# The Macro Neutrality of Exchange-Rate Regimes in the presence of Exporter-Importer Firms

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**Abstract**

I characterize exchange-rate regime breaks for thirty countries between 1960 and 2019, and I establish that while they affect the volatilities of nominal and real exchange rates they do not change the volatilities of other real macroeconomic variables (output, consumption, investment, and net exports). This is true even in countries in which exports and imports represent a large component of gross domestic product. I propose a model with exporter-importer firms which matches the behavior of nominal and real exchange rates and real macroeconomic variables across exchange-rate regimes, even for economies in which the sum of exports and imports exceeds gross domestic product.

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# 1 Introduction

A substantial body of literature in the field of international macroeconomics and finance seeks to assess the impact of various exchange-rate regimes on real macroeconomic variables.[1] The central natural experiment under examination in this body of literature revolves around the upheaval in the US monetary policy regime following the collapse of the Bretton Woods system in 1973. Consequently, the bulk of research findings in this field trace their origins to an exogenous shock that transpired fifty years ago, and affected the United States whose modest volume of international trade in proportion to its overall economic output is an outlier. My paper is the first attempt to provide comprehensive insight into the repercussions of various exchange-rate regimes on real macroeconomic variables.

My approach surmounts the limitations of the conventional paths, as I examine natural experiments beyond the breakdown of the Bretton Woods system. Furthermore, I consider countries with relatively higher volumes of international trade compared to the United States. This perspective contributes a novel dimension to our understanding of these dynamics. To explain the disconnect between the real exchange rate and other real macroeconomic variables in a dynamic stochastic general equilibrium model, it proposes as a theoretical mechanism—grounded in micro empirical evidence in international trade—that exporters are also intensive importers.

My first step is to propose a characterization of exchange-rate regimes based on the trade-weighted exchange rates of thirty countries from 1957 to 2019. For all these economies, I show that structural breaks in the volatility of nominal exchange rates are systematically associated with structural breaks in the volatility of real exchange rates. I can then consider a larger set of exchange-rate regime breaks than the breakdown of the Bretton Woods system.

Second, I document a muted reaction to exchange-rate regime breaks of various real macroeconomic variables (output, consumption, investment, and net exports), but not the real exchange rate. The reaction is muted even though I consider countries that have more exports and imports, compared to total output, than the United States. In the United States, the amount of international trade—that is, exports plus imports—is relatively small compared to total output, with an average trade-to-GDP ratio of about 16% between 1960 and 2019; in contrast, Belgium, one of the countries I consider, has an average trade-to-GDP ratio of about 101% over the same period.

Third, I propose a dynamic stochastic general equilibrium model to demonstrate the muted reaction of various real macroeconomic variables to real–exchange rate movements across exchange-rate regime breaks, assuming international financial-market segmentation, deviations from the law of one price, and the presence of exporter-importer firms. Accounting for sizable exporter-importer firms is key to match the observed muted reaction of real macroeconomic variables to real–exchange rate movements. I do indeed show that the muted reaction cannot be matched quantitatively without considering sizable exporter-importer firms. This is true in the aggregate for countries such as Belgium, but it is also true for countries such as the United States once I restrict the focus to exports and imports.

[1] Examples include Friedman (1953), Mussa (1986), and Baxter and Stockman (1989).

I show that for the United States, the overall muted response results from a mix of a counterfactually large response of exporters and importers, with these firms being a small fraction of the overall economy.

Finally, the paper also provides a contribution at a broader level. The dynamic stochastic general equilibrium model is able to reconnect the macroeconomic empirical regularities from the international macro-finance literature to a theoretical mechanism that is grounded in the micro empirical evidence of international trade, establishing how this micro mechanism is pivotal for the explanation of the macroeconomic empirical regularities.

In the first part of the paper, using trade-weighted exchange rates covering thirty countries from 1957 to 2019, I characterize exchange-rate regimes based on a statistical approach only. In standard bilateral classifications, the definition of exchange-rate regime for a given country relies on its central bank's decision to keep the currency either floating or pegged to a reference currency. Bilateral classifications typically identify exchange-rate regime breaks when one of the two central banks changes its decision and induces a simultaneous volatility break in the bilateral series of nominal and real exchange rates. Instead, I identify the structural breaks in the volatility of the trade-weighted nominal and real exchange rates with the structural break test developed by Lavielle (1999) and Lavielle and Moulines (2000). I find that every break in the set of structural breaks in the volatility of the nominal–exchange rate series corresponds to a structural break in the volatility of the real–exchange rate series.

Second, I consider how real macroeconomic variables react to exchange-rate regime breaks when considering all thirty countries in my sample, which covers sixty-two exchange-rate regime breaks from 1957 to 2019. A robust finding is that the volatilities of all real macroeconomic variables show no statistically significant change across breaks, with a single exception: the real exchange rate. Crucially, this result does not depend on countries' amount of exports and imports. The challenge is to explain why, when a country moves from a pegged to a floating regime, the resulting volatility of the exchange rates is not transmitted to other real macroeconomic variables.

We have to question if we are able to find a set of assumptions that ensure the consistency of a theoretical model with the empirical evidence for a country such as Belgium with an average trade-to-GDP ratio of about 101%. Belgium has a higher average trade-to-GDP ratio than the United States, but it is not an outlier. Czech Republic, Latvia, or Slovenia, for instance have higher average trade-to-GDP ratios.[2] Therefore, finding such a set of assumptions would ensure the generality of the theoretical model for a large set of countries presenting a higher average trade-to-GDP ratios than the United States. I find that three assumptions are sufficient:

1. International financial markets are imperfect, following Gabaix and Maggiori (2015). In any complete-market model, the condition of efficient risk sharing tends to make the consumption difference comove with the real exchange rate. But this result is invalidated by several empirical studies (e.g., Backus and Smith 1993).[3]

[2] See Panels (c) and (d) of Figure 2.

[3] Moreover, this assumption guarantees that the financial element of the model matches a set of empirical facts from the finance literature, the most relevant of which is room for deviations from uncovered interest parity (see Fama 1984).

2. There are deviations from the law of one price in the form of variable markups and local currency pricing, following the empirical industrial-organization literature.[4] Though this assumption helps to improve the fit of theoretical models to the muted reaction of real macroeconomic variables to real–exchange rate movements (e.g., Gabaix and Maggiori 2015), it is not sufficient.

   In the calibration section, using data on Belgium between 1960 and 2019, I show that only assuming imperfect international financial markets and deviations from the law of one price, is not enough to match the observed muted reaction. Additionally, I also show that modelling without sizable exporter-importer firms misses an important feature of the US data: it is unable to capture the muted reaction of either exports or imports to exchange rate movements when they are treated separately. Under a floating regime, exchange rates are highly volatile and exporters are not able to prevent exports from responding to exchange rates, although they can adjust their markups and price in terms of the local currency.

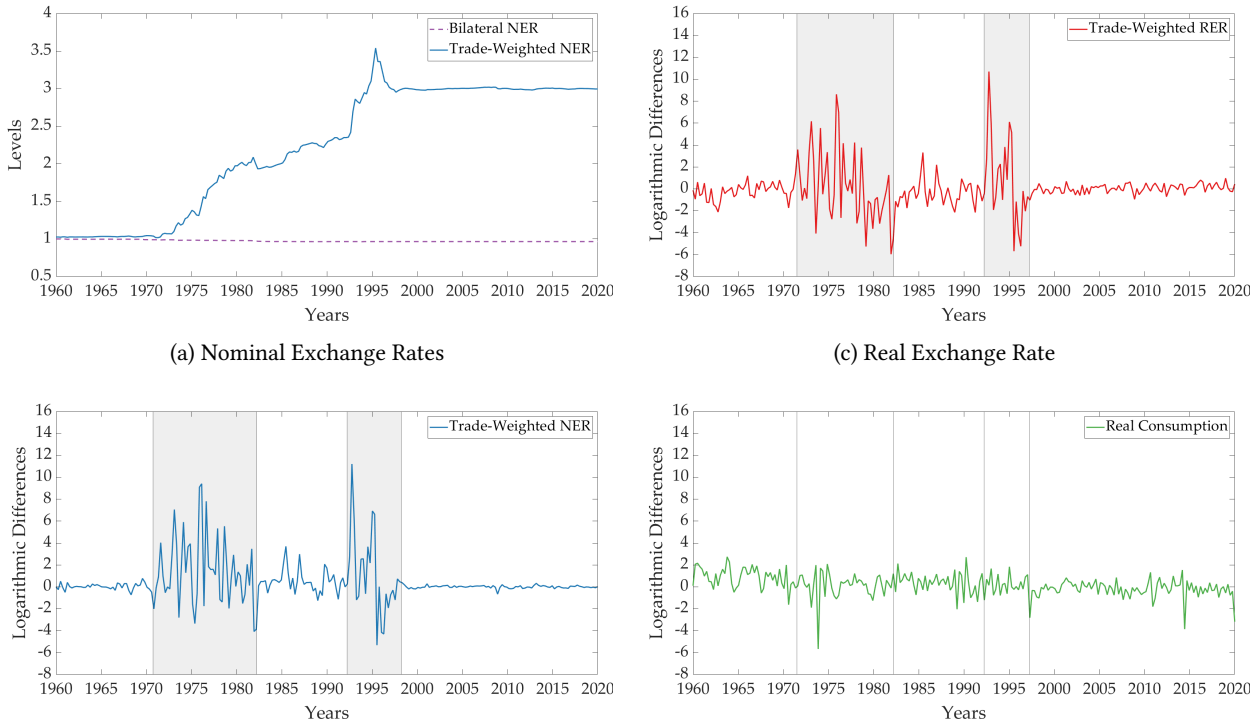
3. Exporters are simultaneously importers, following Amiti, Itskhoki, and Konings (2014), who use Belgian firm-product-level data on imports and exports between 2000 and 2008. This feature is the key ingredient to match the observed muted reaction in an economy with a large amount of exports and imports, compared to total output, in a general equilibrium model. It is not only theoretically appealing but empirically plausible, considering two stylized facts about Belgium in Amiti, Itskhoki, and Konings (2014).

   First, the authors find evidence that 78% of exporters in Belgium also import goods and, more crucially, that the exporters who intensively import goods account for 83% of all Belgian exports. Second, they show that the ratio between imported inputs and exports is 74% for the import-intensive exporters. In other words, the import-intensive exporters account for a disproportionately large share of exports and keep their prices unchanged despite exchange-rate volatility, thanks to the imported inputs in the marginal-cost channel.

   In the calibration section, using data on Belgium between 1960 and 2019, I show that my model with exporter-importer firms can reproduce the comovement of nominal and real exchange rates and the muted reaction of real macroeconomic variables. Additionally, I show that it is able to capture the muted reaction of either exports or imports to exchange rate movements, when they are treated separately, for the United States.

   I introduce exporter-importer firms following Amiti, Itskhoki, and Konings (2014) because this introduction results in a transparent calibration of the import intensity of the exporters in the model. However, such a structure of trade is not peculiar to Belgium and has been documented in other countries, such as France, which have much lower average trade-to-GDP ratios (e.g., Barbiero 2022).

[4] For instance, Goldberg and Verboven (2001, 2005) find not only that the law of one price does not hold, but also that firms absorb the exchange-rate shocks thanks to a local component of their marginal costs and markup adjustment.




(a) Nominal Exchange Rates

(c) Real Exchange Rate

(b) Nominal Exchange Rate

(d) Real Consumption

Figure 1: Exchange Rates and Real Consumption in Belgium (1960-2019)

*Notes for Panels (a) and (b):* The bilateral nominal–exchange rate series between Belgium and Germany is dashed and in magenta, and the trade-weighted nominal–exchange rate series between Belgium and the rest of the world is in blue; I normalize the two series such that they are both equal to 1 in the first quarter of 1960. The vertical lines represent exchange-rate regime breaks identified in the trade-weighted nominal–exchange rate series in logarithmic difference. I shade the periods with floating regimes in the trade-weighted nominal–exchange rate series.
*Notes for Panels (c) and (d):* The trade-weighted real–exchange rate series is in red and the real consumption-difference series is in green. The vertical lines represent exchange-rate regime breaks identified in the trade-weighted real–exchange rate series in logarithmic difference. I shade the periods with floating regimes in the trade-weighted real–exchange rate series.
*Sources:* The Bank of Italy's Exchange Rates Portal, the International Monetary Fund's Direction of Trade Statistics and International Financial Statistics, and the Organisation for Economic Co-operation and Development's OECD.Stat. For details, see Section 2.

Figure 1, depicting time series for Belgium between 1960 and 2019, motivates my work and illustrates the gains deriving from my characterization of exchange-rate regimes based on a statistical approach to trade-weighted exchange rates. Panel (a) plots the bilateral nominal exchange rate between Belgium and Germany, the reference country for the Belgian economy, (dashed) and the trade-weighted nominal exchange rate between Belgium and the rest of the world. Both series are in levels, at a quarterly frequency. Panel (b) plots the trade-weighted nominal exchange rate between Belgium and the rest of the world in logarithmic difference, at a quarterly frequency.

Belgium's regime is typically considered pegged for the entire period in standard bilateral classifications.[5] Indeed, it was pegged if one considers the bilateral nominal exchange rate between Belgium and Germany as shown in Panel (a). But if one considers the trade-weighted nominal exchange rate between Belgium and the rest of the world, the picture completely changes and this is the pivotal advantage of my characterization of exchange-rate regimes.

[5]See Levy-Yeyati and Sturzenegger (2005), Klein and Shambaugh (2010), Ilzetzki, Reinhart, and Rogoff (2019), and Petracchi (2022).

In Panel (b), using the test developed by Lavielle (1999) and Lavielle and Moulines (2000), I identify four structural breaks in volatility that define five exchange-rate regimes: a pegged regime from 1960 to 1970, a floating regime from 1970 to 1982, a pegged regime from 1982 to 1992, a floating regime from 1992 to 1998, and a pegged regime from 1998 to 2019. Crucially, the four exchange-rate regime breaks are present not because of Belgian-German bilateral exchange-rate regime breaks but because of Belgium's trading partners, which experienced exchange-rate regime breaks in relation to Germany; as a consequence, they can be interpreted as shocks exogenous to Belgian monetary policy and economic conditions, offering a setting to identify the effects of different exchange-rate regimes.

Panels (c) and (d) in Figure 1 plot Belgium's trade-weighted real–exchange rate series and its real-consumption-difference series respectively, at a quarterly frequency, between 1960 and 2019. Both series are in logarithmic difference and represent Belgium versus the rest of the world. The trade-weighted real–exchange rate series presents four structural breaks in volatility, corresponding to the four exchange-rate regime breaks in the Belgian trade-weighted nominal exchange rate. Meanwhile, the real-consumption-difference series presents no structural breaks; the shocks do not alter the volatility of the real-consumption-difference series, even in an economy with a large amount of exports and imports compared to total output.

**Related Literature.** My paper contributes to several strands of the literature. First, the characterization of exchange-rate regimes in the context of trade-weighted exchange rates goes beyond the monetary non-neutrality arising from bilateral exchange rates as seen in the Mussa puzzle (Mussa 1986) and its generalization (Petracchi 2022).[6] Second, my analysis of real macroeconomic variables in relation to exchange-rate regime breaks connects to a large literature on exchange rates and macroeconomic outcomes (Friedman 1953, Meese and Rogoff 1983, Fama 1984, Baxter and Stockman 1989, Cole and Obstfeld 1991, Backus and Smith 1993, Obstfeld and Rogoff 2000, Farhi and Gabaix 2015, Lustig and Verdelhan 2019, Amador et al. 2020, Itskhoki and Mukhin 2021, Engel and Wu 2022, and Lilley, Maggiori, and Schreger 2022). My theoretical model is related to the exchange rate portfolio-balance literature (Kouri 1976, Jeanne and Rose 2002, Gabaix and Maggiori 2015, Cavallino 2019, Maggiori 2022, Bacchetta, Benhima, and Berthold 2023, Bacchetta, Davis, and van Wincoop 2023) and the literature that evaluates the effects of different exchange-rate regimes (Monacelli 2004, Benigno and Benigno 2008, Ayres, Hevia, and Nicolini 2021, Flaccadoro and Nispi Landi 2022, and Itskhoki and Mukhin 2022). It is also related to the international-trade literature that tries to explain the lack of correlation between exchange rates and other real macroeconomic variables using firm-product-level imports and exports, in particular Amiti, Itskhoki, and Konings (2014), Barbiero (2022), and Blaum (2022).

[6] Using bilateral time series primarily on the United States and thirteen advanced countries between 1957 and 1984, Mussa (1986) documents what is now referred to as the Mussa Puzzle: the 1973 breakdown of the Bretton Woods system increased the volatility of not only the nominal US-dollar exchange rate but the real US-dollar exchange rate, which implies monetary non-neutrality.

# 2 Empirical Facts and Exchange-Rate Regimes

In Section 2.1, I introduce a characterization of exchange-rate regimes, based on thirty countries from 1957 to 2019, and provide evidence for the Mussa puzzle—the fact that nominal and real exchange rates comove across exchange-rate regimes—in the context of trade-weighted exchange rates. In Section 2.2, I consider real-macro-variable time series—output, consumption, investment, and net exports—of the thirty countries to show that exchange rate disconnect—that is, the muted reaction of real macroeconomic variables to real exchange-rate movements—remains persistent across exchange-rate regimes.[7]

**Data.** I use quarterly data covering the 1957-2019 period for thirty countries—twenty-four European countries and six non-European G20 countries. The choice of the countries for my empirical analysis is exclusively based on the availability of the historical time series for the real macroeconomic variables—output, consumption, investment, and net exports. However, complete data for all the countries are not available and a list of time periods for each country's variable is in Table 7 in Appendix A.2. The twenty-four European countries include the twenty-one European Union member countries along with Norway, Switzerland, and the United Kingdom, while the six non-European G20 countries are Australia, Brazil, Canada, Japan, South Africa, and the United States.[8]

## 2.1 A Characterization of Exchange-Rate Regimes

I begin by constructing trade-weighted exchange rates with a twofold purpose. First, they allow me to empirically evaluate the magnitude of exchange-rate shocks, for any given country in relation to its trading partners, which have to be considered in general equilibrium models. Second, they allow me to introduce a characterization of exchange-rate regimes, where I identify exchange-rate regime breaks through a heteroskedasticity-based approach only.

Monthly time-series data on bilateral nominal exchange rates come from the Exchange Rates Portal of the Bank of Italy; I use the Deutsche Mark as the reference currency for the studied European countries and the US dollar for the non-European G20 countries. I obtain the bilateral nominal–exchange-rate time series for each European country by combining the dollar/Deutsche Mark time series and the dollar/euro time series after December 2001, at which time 1 euro was worth 1.95583 Deutsche Marks, with the various other dollar/foreign-currency time series.[9] I then combine them, using the trade weights from the International Monetary Fund's Direction of Trade Statistics, to obtain trade-weighted nominal exchange rates.[10]

[7] The phrase "exchange rate disconnect" generically refers to the absence of correlation between exchange rates and other macroeconomic variables; see Obstfeld and Rogoff (2000).

[8] The twenty-four studied member countries of the European Union are Austria, Belgium, the Czech Republic, Denmark, Estonia, Finland, France, Germany (West Germany before October 1990), Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, the Netherlands, Poland, Portugal, the Slovak Republic, Slovenia, Spain, and Sweden.

[9] If a currency was renominated—for example, the French franc in January 1960—I normalized the series in order to remove the ensuing jump.

[10] For any given country, I use as weights the mean values of its exports and imports, averaged over the 1957-2019 period, to and from Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

Finally, I combine the latter rates with monthly consumer price indexes (CPIs) from the International Monetary Fund's International Financial Statistics, using the same weights as above, to obtain CPI-based trade-weighted real exchange rates.[11]

Next, I identify the exchange-rate regime breaks by applying the heteroskedasticity-break test, developed by Lavielle (1999) and Lavielle and Moulines (2000), to the first difference of the logarithm of the nominal exchange rate, $\mathcal{E}_t$, and the first difference of the logarithm of the real exchange rate, $\mathcal{Q}_t$, which are defined as follows:

$$\Delta q_t = \Delta e_t + \pi_t^* - \pi_t.$$

Here, $\Delta q_t = ln(\mathcal{Q}_t) - ln(\mathcal{Q}_{t-1})$, $\Delta e_t = ln(\mathcal{E}_t) - ln(\mathcal{E}_{t-1})$, and $\pi_t^* - \pi_t$ is the difference between the inflation rate in the foreign country and the inflation rate in the rest of the world (home country).[12]

The test yields the results for Belgium that are reported in the third column of Table 1. Table 1, together with Table 8 in Appendix A.2.1, which reports the results for all the other studied countries, represents the first main empirical result of this paper. The heteroskedasticity-break test identifies structural breaks in the nominal– and real–exchange-rate series that characterize two types of exchange-rate regime: periods of low exchange-rate volatility (pegged regimes) and periods of high exchange-rate volatility (floating regimes). This characterization of exchange-rate regimes confirms the Mussa puzzle for the reference-currency countries (Germany and the United States) and countries that formally switched their exchange-rate regime from pegged to floating or vice versa (for example, Brazil and the Czech Republic).

It also shows the puzzle for economies that never formally switched in the studied period—for example, Austria and Belgium. For two reasons, this turns out to be crucial for understanding how exchange-rate regimes affect the real economy. First, in the economies that did not switch, the exchange-rate regime breaks are exogenous to their monetary-policy decisions and domestic economic conditions, offering a setting to identify the effects of different exchange-rate regimes.[13] Second, Tables 1 and 8 identify exchange-rate regime breaks, and hence changes from periods of low volatility of the nominal– and real–exchange-rate series to periods of high volatility (and vice versa), for countries for which exports and imports are relatively large compared to total output, offering an ideal setting to test exchange rate disconnect.

[11] For brevity, from here on, I use the phrase "the rest of the world" (home country) to indicate Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States; the term "nominal exchange rate" to refer to the trade-weighted nominal exchange rate; and the term "real exchange rate" to refer to the CPI-based trade-weighted real exchange rate.

[12] A complete description of the Lavielle (1999) and Lavielle and Moulines (2000) test can be found in Appendix A.2.1. The heteroskedasticity-break test does not always identify the structural breaks in the volatility of the nominal– and real–exchange-rate series in the same months since it is very sensitive to observations that significantly depart from the rest.

[13] This a stronger identification strategy than in a standard regression-discontinuity design, in which identification does not rely on the exogeneity of the exchange-rate regime breaks but only requires that potential confounders evolve continuously around the breaks.

Table 1: Belgium's Exchange-Rate Regimes

| Nominal Exchange Rate | Real Exchange Rate | Exchange-Rate Regime |
|---|---|---|
| January 1957 - June 1971 | January 1957 - July 1971 | Pegged Regime |
| July 1971 - September 1978 | August 1971 - January 1982 | Floating Regime |
| October 1978 - July 1992 | February 1982 - July 1992 | Pegged Regime |
| August 1992 - March 1998 | August 1992 - March 1997 | Floating Regime |
| April 1998 - December 2019 | April 1997 - December 2019 | Pegged Regime |

## 2.2 Exchange Rate Disconnect across Exchange-Rate Regimes

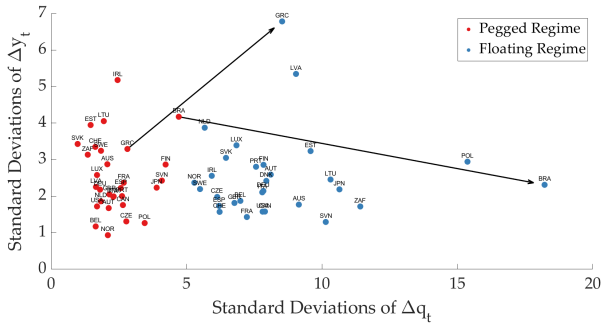
A strand of literature, dating back to Friedman (1953), evaluates the effects of different exchange-rate regimes and asks one of the enduring questions in international macroeconomics and finance: what are the effects of different exchange-rate regimes? Surprisingly, though, it examines principally the breakdown of the Bretton Woods system in 1973, a break in the US exchange-rate regime, and neglects other, similar natural experiments. Figure 2 overcomes this lack of natural experiments, summarizing the empirical results for all the thirty countries and all the exchange-rate regime breaks identified by the heteroskedasticity-break in Section 2.1.

Panel (a) of Figure 2 plots annualized standard deviations of real exchange rates in logarithmic difference, $\sigma(\Delta y_t)$, against annualized standard deviations of real output in logarithmic difference, $\sigma(\Delta q_t)$. These annualized standard deviations are computed across the exchange-rate regimes; the standard deviations are in red (circles) for the pegged regimes and in blue (triangles) for the floating regimes. It is easy to see that when moving from pegged to floating regimes, the volatility of real exchange rates systematically increases for all the studied countries.[14]

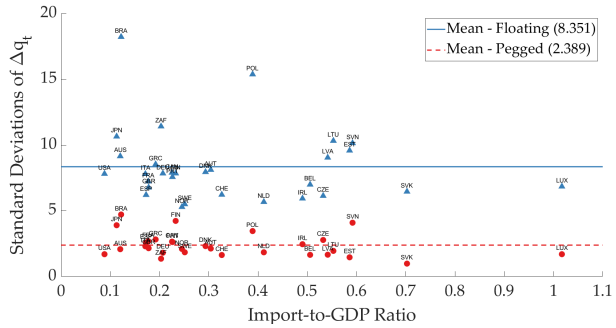
But it is not obvious what happens to output volatility when moving from pegged to floating regimes: for some countries, output volatility increases (for instance, Greece [GRC]); for others, it decreases (for instance, Brazil [BRA]). To offer a more systematic answer, Panel (b) of Figure 2 reports in green, country by country, the differences in $\sigma(\Delta y_t)$ across exchange-rate regimes ($\Delta\left[\sigma(\Delta y_t)\right]$) against the differences in $\sigma(\Delta q_t)$ across exchange-rate regimes ($\Delta\left[\sigma(\Delta q_t)\right]$). Overall, Panel (b) of Figure 2 shows a negative correlation that is not statistically significant.[15]

[14]Here, in order to match the quarterly frequency of the real macroeconomic variables, I identify the exchange-rate regime breaks by applying the heteroskedasticity-break test, developed by Lavielle (1999) and Lavielle and Moulines (2000), to the first difference of the logarithm of the quarterly real exchange rate.

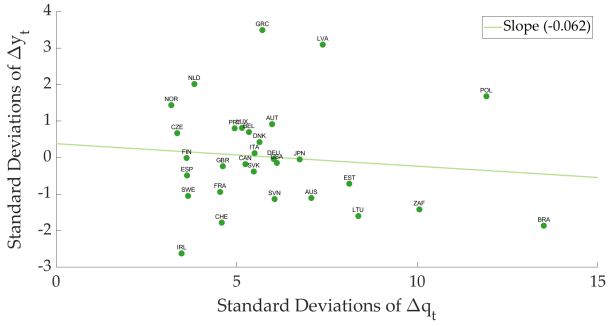
[15]The coefficient of the OLS regression of $\Delta\left[\sigma(\Delta y_t)\right]$ on $\Delta\left[\sigma(\Delta q_t)\right]$ is -0.062 and the 95% confidence interval, using heteroskedasticity-robust standard errors, is [-0.311, 0.188] (the p-value of the test, under the null hypothesis of an OLS coefficient equal to zero, is 0.616).



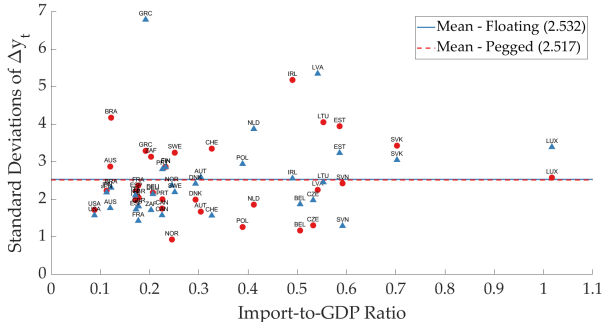

(a) Volatility of Outputs against Volatility of Real Exchange Rates




(c) Volatility of Real Exchange Rates against Countries' Import-to-GDP Ratio




(b) Volatility of Outputs against Volatility of Real Exchange Rates (differences across exchange-rate regimes)




(d) Volatility of Outputs against Countries' Import-to-GDP Ratio

Figure 2: Volatility of Real Exchange Rates and Volatility of Outputs across Exchange-Rate Regimes

*Notes:* The annualized standard deviations are in red (circles) under the pegged regimes and in blue (triangles) under the floating regimes; the differences in the annualized standard deviations across regimes are in green.
*Sources:* The Bank of Italy's Exchange Rates Portal, the International Monetary Fund's Direction of Trade Statistics and International Financial Statistics, and the Organisation for Economic Co-operation and Development's OECD.Stat.

Panels (c) and (d) of Figure 2 expand on this result by plotting $\sigma(\Delta q_t)$ and $\sigma(\Delta y_t)$ against countries' import-to-GDP ratio across exchange-rate regimes.[16] Under the pegged regimes, the standard deviations of $\Delta q_t$ and $\Delta y_t$ are in red (circles); under the floating regimes, in blue (triangles). Panels (c) and (d) document that when one orders the countries by import-to-GDP ratio, moving from a pegged to a floating regime increases mean real–exchange-rate volatility (upper part) without changing mean output volatility (lower part). The characterization of exchange-rate regimes documents that exchange-rate regime breaks are associated with large changes in the volatility of real exchange rates. This result can also be seen in Panel (c) of Figure 2, where we see that moving from pegged to floating regimes increases the mean standard deviation of the real exchange rate by about 350%, from 2.389 to 8.351.

However, this result makes the finding of Panel (d) of Figure 2 much more puzzling with respect to the Mussa puzzle and the US economy, which is represented by the leftmost two points: not only does moving from pegged to floating not change the mean standard deviation of real output across regimes, but it does not systematically increase

[16] For each country, I use as a proxy for the amount of international trade its mean import-to-GDP ratio, for the corresponding time period in Table 7 in Appendix A.2, which is the relevant value to calibrate the openness-to-international-trade parameter $\gamma$ in the theoretical model.

output volatility in economies for which imports are relatively small compared to total output (those in the center and on the left). Moreover, Table 2 reports the OLS coefficients of the regression of $\sigma(\Delta q_t)$ on import-to-GDP ratio and the regression of $\sigma(\Delta y_t)$ on import-to-GDP ratio across exchange-rate regimes. It formally shows that there is no statistically significant correlation between the volatilities of real exchange rates (nor real output) and countries' import-to-GDP ratio across exchange-rate regimes.

Countries experience exchange-rate regime breaks, increasing the volatility of their real exchange rates and hence real shocks to their economies, but do not display systematically increased volatility in their real output; additionally, I find no statistically significant correlation between the volatilities of real exchange rates (nor output) and countries' amount of trade with the rest of the world in either regime (Table 2).

Finally, Table 3 provides some additional details by including other real macroeconomic variables: consumption ($\Delta c_t$), investment ($\Delta z_t$), and net exports ($\Delta nx_t$).[17] Under the pegged regimes, the mean volatility of the real exchange rate is low and at the same order of magnitude as real output's mean volatility, but there is a disconnect under the floating regimes: the floating-pegged ratio for the real exchange rate is about 3.5, but the ratio is around 1 for all the other real macroeconomic variables.

Thus, the second main empirical result of the paper is that exchange rate disconnect remains persistent across exchange-rate regimes, even when countries for which imports, compared to total output, are larger than the United States are studied. The above patterns of change in the volatilities of the real exchange rate and other real macroeconomic variables motivate my theoretical analysis in the next section, which aims to resolve exchange rate disconnect without relying on a country's openness to international trade.

[17] Quarterly time-series data on real output, consumption, investment, and net exports come from the Organisation for Economic Co-operation and Development's OECD.Stat.

Table 2: Relationship between Import-to-GDP Ratio and Volatilities across Exchange-Rate Regimes

| Exchange-Rate Regime | $\sigma(\Delta q_t)$ | $\sigma(\Delta y_t)$ |
|---|---|---|
| Pegged Regime | -1.260<br>[-2.767, 0.246] | 0.721<br>[-0.776, 2.219] |
| Floating Regime | -2.000<br>[-6.082, 2.083] | 1.324<br>[-0.282, 2.930] |

*Notes:* The second column reports the OLS coefficients of the regression of annualized standard deviations of $\Delta q_t$ on import-to-GDP ratio across exchange-rate regimes; the third column reports the OLS coefficients of the regression of annualized standard deviations of $\Delta y_t$ on import-to-GDP ratio across exchange-rate regimes; 95% confidence intervals, using heteroskedasticity-robust standard errors, are in square brackets, and the p-values of the test, under the null hypothesis of an OLS coefficient equal to zero, are 0.098 [pegged regime / $\sigma(\Delta q_t)$], 0.332 [pegged regime / $\sigma(\Delta y_t)$], 0.324 [floating regime / $\sigma(\Delta q_t)$], 0.102 [floating regime / $\sigma(\Delta y_t)$].

Table 3: Volatilities across Exchange-Rate Regimes

| Exchange-Rate Regime | $\sigma(\Delta q_t)$ | $\sigma(\Delta y_t)$ | $\sigma(\Delta c_t)$ | $\sigma(\Delta z_t)$ | $\sigma(\Delta nx_t)$ |
|---|---|---|---|---|---|
| Pegged Regime | 2.389<br>[2.051, 2.726] | 2.517<br>[2.146, 2.889] | 2.320<br>[1.875, 2.766] | 8.904<br>[5.308, 12.500] | 3.844<br>[2.975, 4.713] |
| Floating Regime | 8.351<br>[7.301, 9.401] | 2.532<br>[2.099, 2.965] | 2.806<br>[2.257, 3.356] | 7.996<br>[6.533, 9.458] | 4.231<br>[3.592, 4.872] |
| Floating-Pegged Ratio | 3.5 | 1.0 | 1.2 | 0.9 | 1.1 |

*Notes:* The table reports the mean annualized standard deviations of real macroeconomic variables across exchange-rate regimes; 95% confidence intervals, using heteroskedasticity-robust standard errors, are in square brackets, and the p-values of the test, under the null hypothesis of equal means across exchange-rate regimes, are respectively 0.000, 0.958, 0.165, 0.635, and 0.466.

# 3 Theoretical Framework

My model builds on an international real business cycle model with productivity and financial shocks, and it includes three crucial features: imperfect international financial markets, deviations (in the form of variable markups and local currency pricing) from the law of one price, and exporter-importer firms. Section 3.1 explains the model. Sections 3.2 explains how resolving exchange rate disconnect requires that exporters simultaneously be intensive importers. In Section 3.3, I complement the model-based analysis with the quantitative results from the calibration.

## 3.1 Model

Time is discrete and runs forever: $t = 0, 1, 2, \ldots$ . There are two countries—home (France) and foreign (Belgium, denoted with an asterisk)—each with its own nominal unit of account in which local prices are quoted. The nominal exchange rate $\mathcal{E}_t$ is the price of Belgian francs in French francs: an increase in $\mathcal{E}_t$ corresponds to a nominal devaluation of the home currency (the French franc). The real exchange rate, $\mathcal{Q}_t \equiv (P_t^* \mathcal{E}_t)/P_t$, is the relative consumer price level in the two countries, with $P_t^*$ being the consumer price index in the foreign country and $P_t$ being the consumer price index in the home country. An increase in $\mathcal{Q}_t$ corresponds to a real depreciation of the home currency. Each country's economy is populated by households, two types of firms (domestic firms and exporter-importer firms), and a government.

The countries are symmetric with the exception of their exchange-rate regime: the foreign country always conducts its monetary policy according to a Taylor rule by targeting inflation (a floating regime), while the home country conducts its monetary policy according to a Taylor rule that switches from targeting the nominal exchange rate (a pegged regime) to targeting inflation (a floating regime). In the following description, I focus on the home country.

### 3.1.1 The Home Country

**Households.** There is a continuum of identical households of measure 1. The representative household solves a consumption-savings problem, maximizing its discounted expected utility over final consumption $C_t$ and labor $L_t$:

$$\max_{\{C_t, L_t, Z_t, B_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} L_t^{1+\varphi} \right).$$

Here, $\beta$ is the household discount factor, $\sigma$ is the relative-risk-aversion parameter, and $\varphi$ is the inverse Frisch elasticity of labor supply, subject to the following budget constraint:

$$P_t C_t + P_t Z_t + \frac{B_{t+1}}{R_t} \leq W_t L_t + R_t^K K_t + B_t + \Pi_{Dt} + \Pi_{Et}.$$

Here, $P_t$ is the consumer price index, $Z_t$ is the gross investment in the capital stock $K_t$, $B_t$ is the quantity of the risk-free bond paying out one unit of the home currency at time $t$, $R_t$ is the gross nominal interest rate, $W_t$ is the nominal wage rate, $R_t^K$ is the nominal rental rate of capital, $\Pi_{Dt}$ and $\Pi_{Et}$ are respectively the profits from the domestic firms and the exporter-importer firms. Here, I assume that the representative household in the home country trades only home-currency bonds and owns only home domestic firms and exporter-importer firms.

The within-period consumption expenditure $P_tC_t$, between the home good $C_{Ht}$ and the foreign good $C_{Ft}$, is allocated to minimize expenditure on final consumption $C_t$:

$$P_tC_t = \int_0^1 \left[P_{Ht}(i)C_{Ht}(i) + P_{Ft}(i)C_{Ft}(i)\right] di.$$

Here, $P_{Ht}$ and $P_{Ft}$ are the home-currency prices of the home and foreign goods. Final consumption $C_t$ is implicitly defined by the Kimball (1995) aggregator as follows:

$$\int_0^1 \left[(1-\gamma)g\left(\frac{C_{Ht}(i)}{(1-\gamma)C_t}\right) + \gamma g\left(\frac{C_{Ft}(i)}{\gamma C_t}\right)\right] di = 1.$$

Here, $\gamma$ is the openness-to-international-trade parameter and the function $g(\cdot)$ is increasing and concave with $-g''(1) \in (0,1)$ and $g(1) = g'(1) = 1$. This minimization results in the following demand schedules:

$$C_{Ht}(i) = (1-\gamma)h\left(\frac{P_{Ht}(i)}{\mathcal{P}_t}\right)C_t \qquad \text{and} \qquad C_{Ft}(i) = \gamma h\left(\frac{P_{Ft}(i)}{\mathcal{P}_t}\right)C_t.$$

Here, the function $h(\cdot) = g'^{-1}(\cdot)$ and controls the curvatures of the demand schedules.[18]

The consumer price index $P_t$ and the auxiliary variable $\mathcal{P}_t$ are implicitly defined by the consumption-expenditure equation and by the Kimball (1995) aggregator, after substituting the home demand schedules:

$$P_t = \int_0^1 \left[(1-\gamma)P_{Ht}(i)h\left(\frac{P_{Ht}(i)}{\mathcal{P}_t}\right) + \gamma P_{Ft}(i)h\left(\frac{P_{Ft}(i)}{\mathcal{P}_t}\right)\right] di, \tag{1}$$

$$\int_0^1 \left\{(1-\gamma)g\left[h\left(\frac{P_{Ht}(i)}{\mathcal{P}_t}\right)\right] + \gamma g\left[h\left(\frac{P_{Ft}(i)}{\mathcal{P}_t}\right)\right]\right\} di = 1. \tag{2}$$

$Z_t$ accumulates according to the following rule—quadratic capital adjustment costs—with depreciation $\delta$ and capital adjustment cost $\kappa$:

$$K_{t+1} = (1-\delta)K_t + \left[Z_t - \frac{\kappa}{2}\frac{(\Delta K_{t+1})^2}{K_t}\right].$$

[18] In this setting, the point elasticity $\theta = -h'(1)$, whereas the constant-elasticity-of-substitution aggregator, with elasticity of substitution $\theta$, is a special case of the Kimball (1995) aggregator when $g(x) = 1 + \frac{\theta}{\theta-1}\left(x^{1-\frac{1}{\theta}} - 1\right)$.

Gross investment $Z_t$ is a bundle of domestic and foreign varieties, as final consumption $C_t$, aggregated according to an analogous Kimball (1995) aggregator and demanded according to analogous demand schedules.

**Domestic firms.** There is a continuum of identical domestic firms of measure 1. The representative domestic firm $i$ produces using a Cobb-Douglas technology with labor $L_{Dt}$, capital $K_{Dt}$, and intermediate inputs $X_{Dt}$:

$$Y_{Ht} = \left(e^{a_t} K_{Dt}^{\vartheta} L_{Dt}^{1-\vartheta}\right)^{1-\phi} X_{Dt}^{\phi}.$$

Here, $a_t$ is the logarithm of total factor productivity, which follows an AR(1) process:

$$a_t = \rho_a a_{t-1} + \sigma_a \epsilon_t^a, \qquad \epsilon_t^a \sim \mathcal{N}(0,1).$$

Here, the persistent parameter $\rho_a \in [0,1]$ and the volatility of the innovation $\sigma_a \geq 0$. The intermediate input $X_{Dt}$ is a bundle of domestic and foreign varieties, like final consumption $C_t$ and gross investment $Z_t$, aggregated according to an analogous Kimball (1995) aggregator and demanded according to analogous demand schedules.

The associated marginal cost of production for the domestic firm is

$$MC_{Dt} = \frac{1}{\varpi}\left[e^{-a_t} R_t^{K^{\vartheta}} W_t^{1-\vartheta}\right]^{1-\phi} P_t^{\phi}, \qquad \text{where} \qquad \varpi \equiv \phi^{\phi}\left[(1-\phi)\vartheta^{\vartheta}(1-\vartheta)^{1-\vartheta}\right]^{1-\phi}.$$

In serving the home market, the domestic firm maximizes profits,

$$\Pi_{Dt}(i) = \left(P_{Ht}(i) - MC_{Dt}\right) Y_{Ht}(i),$$

by optimally setting $P_{Ht}(i)$. Thanks to the Kimball (1995) aggregator, such profit maximization results in variable-markup pricing with a common price across all domestic firms $i$:

$$P_{Ht}(i) = P_{Ht} = \mu\left(\frac{P_{Ht}}{\mathcal{P}_t}\right) MC_{Dt}. \tag{3}$$

Here, the markup function $\mu(x) = \frac{-\frac{\partial \ln h(x)}{\ln x}}{-\frac{\partial \ln h(x)}{\ln x}-1}$ is derived from the demand schedules of $C_t$, $K_t$, and $X_t$ in the home country. The aggregate profits, $\Pi_{Dt} = \int_0^1 \Pi_{Dt}(i)di$, are distributed to the households.

**Exporter-importer firms.** There is a continuum of exporter-importer firms of measure 1. The representative exporter-importer firm $j$ still produces using a Cobb-Douglas technology with labor $L_{Et}$, capital $K_{Et}$, and intermediate inputs $X_{Et}$ but also directly imports inputs $E_{Ft}^*$, priced in the foreign currency, from the foreign country:

$$Y_{Ht}^* = \left[\left(e^{a_t} K_{Et}^{\vartheta} L_{Et}^{(1-\vartheta_t)}\right)^{1-\phi} X_{Et}^{\phi}\right]^{1-\phi^e} \left(E_{Ft}^*\right)^{\phi^e}.$$

Given the foreign-currency price of the foreign good $P^*_{Ft}$, the associated marginal cost of production for the exporter-importer firm is

$$MC_{Et} = \frac{1}{\varpi^e}\left\{\left[e^{-a_t}R_t^{K^{\vartheta}}W_t^{1-\vartheta}\right]^{1-\phi}P_t^{\phi}\right\}^{1-\phi^e}(\mathcal{E}_t P^*_{Ft})^{\phi^e}, \quad \text{where}$$

$$\varpi^e \equiv \phi^{e\phi^e}\left\{(1-\phi^e)\phi^{\phi}\left[(1-\phi)\vartheta^{\vartheta}(1-\vartheta)^{1-\vartheta}\right]^{1-\phi}\right\}^{1-\phi^e}.$$

In serving the foreign market, the exporter-importer firm maximizes profits,

$$\Pi_{Et}(j) = (P^*_{Ht}(j)\mathcal{E}_t - MC_{Et})\,Y^*_{Ht}(j),$$

by optimally setting $P^*_{Ht}(j)$. Thanks to the Kimball (1995) aggregator, such profit maximization results in variable-markup pricing with a common price across all exporter-importer firms $j$:

$$P^*_{Ht}(j) = P^*_{Ht} = \mu\left(\frac{P^*_{Ht}}{\mathcal{P}^*_t}\right)\frac{MC_{Et}}{\mathcal{E}_t}. \tag{4}$$

Here, the markup function $\mu(x) = \frac{-\frac{\partial \ln h(x)}{\ln x}}{-\frac{\partial \ln h(x)}{\ln x}-1}$ is derived from the demand schedules of $C^*_t$, $K^*_t$, and $X^*_t$ in the foreign country. The aggregate profits, $\Pi_{Et} = \int_0^1 \Pi_{Et}(j)dj$, are distributed to the households.

**Government in the home country.** The fiscal authority is fully passive, in the sense that I abstract from government spending and taxation, whereas the monetary authority conducts monetary policy according to the following Taylor rule:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)[\omega_\pi \pi_t + \omega_e(e_t - \bar{e})].$$

Here, $i_t = ln(R_t)$, $\bar{e}$ is the logarithm of the targeted nominal exchange rate, $0 \geq \rho_i \leq 1$, $\omega_\pi > 1$, and $\omega_e \geq 0$. The parameter $\rho_i$ represents interest rate smoothing in the monetary-policy rule, whereas the parameters $\omega_\pi$ and $\omega_e$ respectively represent the weights of the two monetary-policy objectives, inflation targeting and nominal-exchange-rate targeting. When $\omega_e = 0$, the monetary authority implements a floating regime; when $\omega_e > 0$, a pegged regime.

### 3.1.2 The Foreign Country

The foreign country is fully symmetric to the home country except that the monetary authority conducts monetary policy according to the following Taylor rule:

$$i^*_t = \rho_{i^*} i^*_{t-1} + (1-\rho_{i^*})\omega_{\pi^*}\pi^*_t.$$

Here, $0 \geq \rho_{i^*} \leq 1$ and $\omega_{\pi^*} > 1$. The parameter $\rho_{i^*}$ represents interest rate smoothing in the monetary-policy rule. Unlike in the home country, the monetary authority always implements a floating regime.

#### 3.1.3 International Financial Markets

The international financial markets are segmented as in Gabaix and Maggiori (2015) since the home and foreign households cannot directly trade any bonds with each other. Their international financial positions are intermediated by a unit mass of global financial firms, each managed by a financier.

The representative financier solves the following constrained problem:

$$\max_{Q_t} V_t = \mathbb{E}_t \left[ \beta (R_t - R_t^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}) \right] Q_t, \quad \text{subject to} \quad V_t \geq \Gamma_t \frac{Q_t^2}{\mathcal{E}_t}.$$

Here, $Q_t$ is the balance-sheet position of the financier, in French francs, and $\Gamma_t = \xi \left[ \mathbb{V}\text{ar}_t(\mathcal{E}_{t+1}) \right]^\alpha$, with $\xi \geq 0$ and $\alpha \geq 0$. $\Gamma_t$ represents the financiers' risk-bearing capacity. For simplicity of the model, I assume that financiers rebate their profits and losses to the foreign households, not the home ones.[19]

An important assumption of the model is that the representative household in the home (foreign) country trades only home-currency (foreign-currency) bonds and owns only home (foreign) domestic firms and exporter-importer firms. As a consequence, the home (foreign) country is borrowing and lending in its own currency only. One can alternatively write the model with a representative household (and representative domestic firm and exporter-importer firm) which can borrow in the foreign currency, stipulating an additional channel of transmission of nominal exchange-rate shocks in the same fashion of Fukui, Nakamura, and Steinsson (2023). European countries up to the beginning of the nineties had tight capital control restrictions on foreign currency exposures, so I take this simpler formulation.

#### 3.1.4 Market Clearing

**Labor and capital markets.** In the home and foreign countries, nominal wage rates $W_t$ and $W_t^*$ adjust to clear the home and foreign labor markets, respectively, and nominal rental rates of capital $R_t^K$ and $R_t^{K*}$ adjust to respectively clear the home and foreign capital markets, respectively.

**Goods market.** In the home country, clearing the goods market requires that total production by the home domestic firms and exporters is split between supply to the home and foreign markets respectively and satisfies the

[19] I introduce exogenous financial shocks to the international financial markets only in the linearized version of the model, without taking a stance on their microfoundation, as they can be equally generated from exogenous portfolio flows of the households, as in Gabaix and Maggiori (2015); from noise traders, as in Itskhoki and Mukhin (2021, 2022); or from biased exchange-rate expectations, as in Jeanne and Rose (2002). Akinci, Şebnem Kalemli-Özcan, and Queralto (2023) also emphasizes intermediation frictions in the presence of long-lived financial intermediaries that face leverage constraints.

demand in each market:

$$Y_t = Y_{Ht} + Y^*_{Ht},$$

$$Y_{Ht} = C_{Ht} + X_{Ht} + Z_{Ht} + E_{Ht} = (1-\gamma)h\left(\frac{P_{Ht}}{\mathcal{P}_t}\right)[C_t + X_t + Z_t] + E_{Ht}, \text{ and}$$

$$Y^*_{Ht} = C^*_{Ht} + X^*_{Ht} + Z^*_{Ht} = \gamma h\left(\frac{P^*_{Ht}}{\mathcal{P}^*_t}\right)[C^*_t + X^*_t + Z^*_t].$$

Finally, I derive the home country's budget constraint:

$$\frac{B_{t+1}}{R_t} - B_t = NX_t \qquad \text{with} \qquad NX_t = (\mathcal{E}_t P^*_{Ht} Y^*_{Ht} + P_{Ht} E_{Ht}) - (P_{Ft} Y_{Ft} + \mathcal{E}_t P^*_{Ft} E^*_{Ft}).$$

Here, $NX_t$ are net exports in units of the home currency.

Net exports contain two extra terms, relative to a model without exporter-importer firms: the directly imported inputs of the foreign exporter-importer firm ($E_{Ht}$) and the directly imported inputs of the home exporter-importer firm ($E^*_{Ft}$), which are priced in home and foreign currencies, respectively.

**International financial markets.** Clearing the international financial markets requires that the balance sheet position of the financiers in French francs $Q_t$ equals $B_t$ and the balance sheet position of the financiers in Belgian francs $Q^*_t$ equals $B^*_t$.

#### 3.1.5 Equilibrium Definition and Model Solution

In Appendix A.3.1.5, I define an equilibrium in the nonlinear model. I solve the model by logarithmic linearization around a symmetric steady state with steady-state markup $\bar{\mu} = 1$ and, from now on, I denote all the expressions in terms of deviations from the symmetric steady-state equilibrium; for example, $y_t \equiv \ln(Y_t) - \ln(\bar{Y})$.

### 3.2 Exporter-Importer Firms Resolving Exchange Rate Disconnect

Two equations characterize the linearized model around a symmetric steady state: the modified UIP condition in the international financial markets, and the home flow budget constraint.

The logarithmic linearization of the equilibrium condition in the international financial markets results in the following modified UIP condition, which is subject to exogenous financial shocks:

$$i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} = \chi_1 \psi_t - \chi_2 b_{t+1}. \tag{5}$$

Here, $b_{t+1} = B_{t+1}/\bar{Y}$, $\chi_1 = \frac{\Gamma}{\beta}$, and $\chi_2 = \bar{Y}\Gamma$. The exogenous financial shocks $\psi_t$ follow an AR(1) process:

$$\psi_t = \rho_\psi \psi_{t-1} + \sigma_\psi \epsilon_t^\psi, \qquad \epsilon_t^\psi \sim \mathcal{N}(0,1).$$

The persistent parameter $\rho_\psi \in [0,1]$, and the volatility of the innovation $\sigma_\psi \geq 0$.[20] When the financiers' risk-bearing capacity $\Gamma = 0$, they can absorb any imbalances, which results in no deviation from the UIP condition; $i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = 0$. The higher the $\Gamma$—that is, the lower the financiers' risk-bearing capacity—the more segmented the international financial markets. For $0 < \Gamma < \infty$, the model endogenously generates UIP deviations.[21]

The logarithmic linearization of the home country's flow budget constraint results in the following equation:

$$nx_t = (1-\phi^e)\tilde{\gamma} e_t + \tilde{\gamma}(y_{Ht}^* - y_{Ft} + p_{Ht}^* - p_{Ft}) + \phi^e \tilde{\gamma}(e_{Ht} - e_{Ft}^* + p_{Ht} - p_{Ft}^*). \tag{6}$$

Here, $nx_t = \frac{NX_t}{\bar{Y}}$ and $\tilde{\gamma} \equiv \frac{\gamma}{1+\phi^e\gamma}$.

Thanks to the inclusion of exporter-importer firms, I can state the following proposition on how to resolve exchange rate disconnect under the floating regime.[22] I relegate the quantitative analysis to Section 3.3.

**PROPOSITION.**

Assume that $\Gamma > 0$ and $\omega_e = 0$. For any value of $\gamma$, $c_t - c_t^* = \frac{(1+\varphi)}{1+\varphi\sigma}(a_t - a_t^*)$ if $\phi^e \to 1$.

**Proof.** See Appendix A.3.2.

**Discussion.** Here, I show how my model's feature contributes to the literature with the aid of three crucial parameters: $\Gamma$, the financiers' risk-bearing capacity; $\gamma$, the openness-to-international-trade parameter; and $\phi^e$, the import intensity of the exporters.

*Monacelli (2004).* If the financiers' risk-bearing capacity $\Gamma = 0$, the financiers are able to absorb any imbalances, resulting in no deviation from the UIP condition. The model collapses to a model without financial-market frictions, similar to Monacelli's (2004) model in which the Backus and Smith (1993) condition of efficient international risk sharing holds and the consumption difference across countries comoves with the real exchange rate. This model outcome is empirically implausible because of the absence of simultaneous structural breaks in the consumption-difference volatility.[23]

*Itskhoki and Mukhin (2021, 2022).* $\Gamma > 0$ with $\gamma \to 0$ is the solution adopted by Itskhoki and Mukhin (2021,

[20] See footnote 19.

[21] If $\Gamma \uparrow \infty$, the financiers are unwilling to absorb any imbalances; that is, they do not take any positions in the international financial markets.

[22] I state the proposition for the home country; a symmetric one applies for the foreign country.

[23] However, if one introduces price stickiness à la Calvo (1983), the model is able to match the fact that nominal and real exchange rates comove across exchange-rate regime breaks.

2022). In this world, $\phi^e = 0$, the exporters are not intensive importers, and their production technology is identical to domestic firms'. Equation (6) becomes equal to the following:

$$nx_t = \gamma(e_t + y^*_{Ht} - y_{Ft} + p^*_{Ht} - p_{Ft}). \tag{7}$$

Equation (7) illustrates how the openness-to-international-trade parameter plays a crucial role in isolating the exchange-rate volatility in the home economy under the floating regime. This is because if $\gamma \to 0$, as is true for the US economy, real macroeconomic variables do not react ($\gamma = 0$ represents complete autarky). As we will see in Section 3.3, this resolution under the floating regime does not work for Belgium, since real macroeconomic variables strongly react when the openness-to-international-trade parameter $\gamma > 0$. In the case of the United States with $\gamma \to 0$, it is, moreover, unable to capture the muted reaction of exports $y^*_{Ht}$ and imports $y_{Ft}$, taken into account separately, to exchange-rate movements.

*Exporter-importer firms.* Incorporating exporter-importer firms is my main theoretical finding, as it allows me to account for economies for which exports and imports are big compared to total output, with exporter-importer firms à la Amiti, Itskhoki, and Konings (2014): $\Gamma > 0$, $\gamma > 0$, and $\phi^e > 0$.

Under the pegged regimes, the resolution of exchange rate disconnect is straightforward and does not rely on the exporter-importer firms. Suppose that the home country's monetary authority implements a perfect currency board, implying that $e_t = \bar{e}$ for any $t$. Then the financiers' risk-bearing capacity $\Gamma = 0$ and there are no deviations from the UIP condition, so $i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} = i_t - i^*_t - \bar{e} + \bar{e} = 0$. Consequently, real macroeconomic variables are not affected by exchange-rate volatility, which is absent because $e_t = \bar{e}$ for any $t$, but only by productivity shocks.[24]

Under the floating regimes, the resolution of exchange rate disconnect is more complex and crucially relies on exporter-importer firms. Suppose that the home country's monetary authority implements a fully floating regime such that $\omega_e = 0$. Then the financiers' risk-bearing capacity $\Gamma > 0$, implying endogenous UIP deviations and a decreasing capacity to bear the risk of an increasing volatility of $e_t$ because $\Gamma = \xi \left[\mathbb{V}\text{ar}_t(\mathcal{E}_{t+1})\right]^\alpha$. Now, suppose that $\phi^e > 0$, the exporters are intensive importers, and their production technology is very different from domestic firms', as they largely take advantage of imported inputs. The exporter-importer firms then are playing an active role in isolating the exchange-rate volatility in the home economy, *independently* of the openness-to-international-trade parameter $\gamma$. This can be seen in equation (6), in which real macroeconomic variables are increasingly muted to the volatility of $e_t$ for $\phi^e > 0$ and become completely isolated in the limit as $\phi^e \to 1$.[25]

If output is produced by a unique firm that sells in the domestic and foreign markets, the firm has no incentive to specialize its production to serve one of the two markets, in particular the foreign one. So the firm cannot hedge an eventual exchange-rate shock: either it is transmitted to the final consumer through a different price, or it is absorbed

[24] Indeed, the first term on the right-hand side of the home country's flow budget constraint, equation (6), is a constant.
[25] Indeed, the exporter-importer firms use no local inputs if $\phi^e = 1$.

through the firm's markup. However, if output is produced by two types of firms, one selling in the domestic market and the other selling in the foreign market, the latter firm—the exporter—has an incentive to specialize its production to serve the foreign market and import a large part of its inputs from the foreign country. This results in an exporter-importer firm that can hedge the eventual exchange-rate shock, independently of its magnitude, without transmitting it to the rest of the economy.

## 3.3 Calibration

For a transparent comparison between my model with exporter-importer firms and a model with no exporter-importer firms, I follow the assumptions and calibration in Itskhoki and Mukhin (2021, 2022), which presents a model without exporter-importers firms. I adopt the same model parameters, as summarized in Table 9 in Appendix A.3.3, with three exceptions: first, I change the openness-to-international-trade parameter $\gamma$ from 0.07 (the US calibration in Itskhoki and Mukhin 2021) to 0.25 to be consistent with the average imports-to-GDP ratio of Belgium over the 1960–2019 period; second, I modify the capital-adjustment-cost parameter $\kappa$ to match the relative volatility of investment and output, $std(\Delta z_t)/std(\Delta y_t)$, whose value is 2.5 as in Itskhoki and Mukhin (2021); third, I choose 0.23 as the value of $\omega_e$, the weight of nominal–exchange-rate targeting (in the pegged regime) in the Taylor rule of the home country, following Itskhoki and Mukhin (2022), as Itskhoki and Mukhin (2021) do not analyze the pegged regimes. Lastly, I set $\phi^e = 0.74$ following the empirical finding in Amiti, Itskhoki, and Konings (2014) that 74% is the ratio between imported inputs and exports for import-intensive exporters.[26]

My model, like the multi-shock version of Itskhoki and Mukhin's (2021) model, features three exogenous shocks for which I need to calibrate the covariance matrix: two country-specific productivity shocks $(a_t, a_t^*)$ and a financial shock ($\psi_t$). I assume that $\psi_t$ is orthogonal to $(a_t, a_t^*)$, whereas $a_t$ and $a_t^*$ have the same variance (that is, $\sigma_a = \sigma_{a^*}$), and a nonzero correlation ($\rho_{a,a^*}$). I always choose the relative volatility of the shocks, $\sigma_a/\sigma_\psi$, to match the Backus-Smith correlation between the United States and the rest of the world, $corr(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.4$, while I always set the cross-country correlation of productivity shocks, $\rho_{a,a^*}$, to match the correlation of the United States with the rest of the world, $corr(\Delta y_t, \Delta y_t^*) = 0.35$.

### 3.3.1 Calibration Results

**Floating regime.** I find four main results. First, the real exchange rate is strongly correlated with the nominal exchange rate in both models. Second, for values of $\gamma \geq 0.25$, there is no longer disconnect between exchange rates and other real macroeconomic variables in the theoretical model of Itskhoki and Mukhin (2021), whereas my model

[26]Remarkably, this represents, at most, a conservative value of $\phi_e$, as Amiti, Itskhoki, and Konings (2014) use Belgian firm-product-level data on exports and imports between 2000 and 2008, during which Belgium features a pegged regime under my characterization of exchange-rate regimes. Indeed, if one excludes from the calculation exports and imports to and from the euro area, the ratio between imported inputs and exports for the import-intensive exporters becomes 1.44. However, I set $\phi^e = 0.74$ in my calibration, as Belgium was still pegged to some countries (for example, Germany, Austria, the Netherlands) under the floating regimes.

maintains the disconnect thanks to the exporter-importer firms that actively hedge the exchange-rate shocks.[27] Third, price and wage stickiness à la Calvo (1983) does not increase the quantitative fit of my model. Finally, I show that the theoretical model of Itskhoki and Mukhin (2021) also misses an important feature for the United States: it is unable to capture the muted reaction of exports or imports to exchange-rate movements when they are treated separately, while I show that my model can capture this.

Table 4 reports the simulation results, for 10,000 simulations of 120 quarters, and compares the results of my model under the floating regime (that is, $\omega_e = 0$) with the results of the authors' preferred version of the Itskhoki and Mukhin (2021) model, the one with price and wage stickiness à la Calvo (1983). I choose values for $\kappa$, $\sigma_a$, and $\rho_{a,a^*}$ to match $std(\Delta z_t)/std(\Delta y_t) = 2.5$, $corr(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.4$, and $corr(\Delta y_t, \Delta y_t^*) = 0.35$, respectively.

In the first part of Table 4, I set the three values to target the moments in the Itskhoki and Mukhin (2021) model with $\gamma = 0.07$; in the second part, I set the values to target the moments in that model with $\gamma = 0.25$; in the third part, I set them to target the moments in my model, which does not feature price and wage stickiness, with $\gamma = 0.25$. Additionally, I propose a version of my model—in the fifth column of Table 4—that features price and wage stickiness à la Calvo (1983) with the same stickiness parameters of the model in Itskhoki and Mukhin (2021). I do this to make the two models more comparable and to emphasize the pivotal role of exporter-importer firms.

The first result in Table 4 is that both models match the real exchange rate's strong correlation with the nominal exchange rates in all three calibrations. However, while for $\gamma = 0.07$ the Itskhoki and Mukhin (2021) theoretical model can capture the disconnect between the volatility of exchange rates and the other real macroeconomic variables, it loses this capability when $\gamma$ is greater than or equal to 0.25. For any of the three calibrations, my model performs better than the other in insulating the real macroeconomic variables from exchange-rate volatility. This is because of the role of the exporter-importer firms, which actively hedge the exchange-rate shocks, independently of the shocks' magnitude, thanks to their amount of directly imported inputs. When moments are targeted under $\gamma = 0.25$, my model requires a much lower value for $\sigma_a$ (namely, $\sigma_a = 2.9$; see lower part of Table 4) than the Itskhoki and Mukhin (2021) model (namely, $\sigma_a = 5$; see middle part of Table 4). The Itskhoki and Mukhin (2021) model, with no exporter-importer firms, can match a $corr(\Delta q_t, \Delta c_t - \Delta c_t^*)$ of -0.4 only by means of high volatility in exogenous productivity shocks. However, the fifth and sixth columns of Table 4 show that my model performs better without price and wage stickiness à la Calvo (1983), which represents another key difference from the Itskhoki and Mukhin (2021) model, the authors' preferred version of which assumes price and wage stickiness à la Calvo (1983). This is not surprising given that price stability in my model endogenously arises as a result of the capacity of exporter-importer firms to be shock absorbers, thanks to local-currency pricing and their directly imported inputs.[28]

[27] Indeed, the muted reaction of real macroeconomic variables in the theoretical model of Itskhoki and Mukhin (2021, 2022) arises thanks to a calibration that targets the US economy, a country in which exports and imports are relatively small compared to total output. Itskhoki and Mukhin (2021, 2022) discuss other examples of economies with larger amounts of exports and imports compared to total output, than the United States, but they do not consider economies, in which the sum of exports and imports is more than 100% of gross domestic product.

[28] My conjecture is that this latter result would drastically change in the presence of additional exogenous shocks. Indeed, a result in the

Last, Table 5 reports the simulation results, for 10,000 simulations of 120 quarters, and compares the quantitative results of my model under the floating regimes with the quantitative results of the authors' preferred version of the Itskhoki and Mukhin (2021) model taking into account exports and imports separately for the United States. I again set $\kappa$, $\sigma_a$, and $\rho_{a,a^*}$ to target the moments in the Itskhoki and Mukhin (2021) model with $\gamma = 0.07$.

If net exports are decomposed in exports and imports, the openness-to-international-trade parameter $\gamma$ cannot play a role anymore in the model in isolating the exchange-rate volatility; see equations (6) and (7). Indeed, if one takes them separately, their volatility in the Itskhoki and Mukhin (2021) model has the same order of magnitude of the real exchange rate (see the second and third cells in the third column), a result that is at odds with the empirical evidence (see the second and third cells in the second column).

Introducing exporter-importer firms into the model also solves this issue, even without modifying the calibration, because it creates a natural hedging mechanism through local-currency pricing and their directly imported inputs, making real exports and imports insulated to exchange-rate movements.

Overall, there are two key takeaways from these calibration results under the floating regime: (i) the Itskhoki and Mukhin (2021) model, a model without exporter-importer firms, cannot replicate the exchange rate disconnect for a value of the openness-to-international-trade parameter $\gamma \geq 0.25$, and it cannot capture the volatility of exports and imports taken into account separately for a value of the openness-to-international-trade parameter $\gamma = 0.07$; (ii) the same model can be modified by introducing the key feature of exporter-importer firms, which fixes both issues.

**Pegged regime.** Table 6 shows that my model can also accommodate a pegged regime for a value of $\omega_e = 0.23$, without recalibrating the covariance matrix of exogenous shocks. This results in decreased output and consumption volatilities relative to the exchange-rate volatility, but the correlation between the nominal and the real exchange rate is still strong, confirming my model's ability to replicate the Mussa puzzle.

However, it looks like the model quantitatively underperforms, in the pegged regime, in replicating the same moments as before.[29] This is because the real macroeconomic variables' volatility is too low, which can be easily understood in light of my discussion in Section 3.2. Under pegged regimes, countries feature only two exogenous shocks—the country-specific productivity shocks $(a_t, a_t^*)$—as the financial shock, $\psi_t$, is completely absorbed by the financiers, which have full risk-bearing capacity (that is, $\Gamma = 0$) under the pegged regimes.

I can improve on this by adding a third type of shock—a preference shock—to the model, as in Itskhoki and Mukhin (2022), and recalibrating the covariance matrix of exogenous shocks under the floating regime. Nevertheless, as my goal is to explain exchange rate disconnect and the Mussa puzzle, with exporter-importer firms playing a key role in preventing transmission of exchange-rate volatility to the rest of the economy under the floating regime, I do not include preference shocks, as it keeps my model fully comparable with Itskhoki and Mukhin's (2021) model.

literature on dynamic stochastic general equilibrium is that price and wage stickiness can improve the quantitative fit when preference, monetary, or investment-specific shocks are incorporated (see, for instance, Smets and Wouters, 2003, 2007 and Justiniano, Primiceri, and Tambalotti, 2010).

[29] This can be seen by looking at $\sigma(\Delta nx_t)/\sigma(\Delta q_t)$.

Table 4: Main Quantitative Results without and with Exporter-Importer Firms

| Floating Regime ($\omega_e = 0$) | | No Exporter-Importer Firms ($\phi^e = 0$) | | Exporter-Importer Firms ($\phi^e = 0.74$) | | |
|---|---|---|---|---|---|---|
| | US Moments | United States | Belgium | Belgium | Belgium | Belgian Moments |
| | | $\gamma = 0.07$ | $\gamma = 0.25$ | $\gamma = 0.25$ | $\gamma = 0.25$ | |
| $corr(\Delta e_t, \Delta q_t)$ | 0.99 | 1.00 | 0.99 | 1.00 | 1.00 | 0.95 |
| $\sigma(\Delta e_t)/\sigma(\Delta y_t)$ | 5.2 | 3.38 | 1.74 | 2.40 | 2.99 | 3.7 |
| $\sigma(\Delta e_t)/\sigma(\Delta c_t)$ | 6.3 | 5.77 | 4.24 | 5.06 | 5.14 | 4.8 |
| $\sigma(\Delta nx_t)/\sigma(\Delta q_t)$ | 0.10 | 0.17 | 0.59 | 0.29 | 0.28 | 0.17 |
| $corr(\Delta e_t, \Delta q_t)$ | 0.99 | 1 | 0.99 | 1.00 | 1.00 | 0.95 |
| $\sigma(\Delta e_t)/\sigma(\Delta y_t)$ | 5.2 | 1.63 | 1.08 | 1.31 | 1.55 | 3.7 |
| $\sigma(\Delta e_t)/\sigma(\Delta c_t)$ | 6.3 | 3.01 | 2.26 | 2.57 | 2.78 | 4.8 |
| $\sigma(\Delta nx_t)/\sigma(\Delta q_t)$ | 0.10 | 0.16 | 0.59 | 0.30 | 0.28 | 0.17 |
| $corr(\Delta e_t, \Delta q_t)$ | 0.99 | 1.00 | 0.99 | 1.00 | 0.98 | 0.95 |
| $\sigma(\Delta e_t)/\sigma(\Delta y_t)$ | 5.2 | 3.18 | 1.70 | 2.30 | 2.32 | 3.7 |
| $\sigma(\Delta e_t)/\sigma(\Delta c_t)$ | 6.3 | 5.06 | 3.76 | 4.42 | 4.85 | 4.8 |
| $\sigma(\Delta nx_t)/\sigma(\Delta q_t)$ | 0.10 | 0.17 | 0.60 | 0.30 | 0.28 | 0.17 |
| Price and Wage Stickiness | — | YES | YES | YES | NO | — |

*Notes:* The US moments in the second column are from Tables 3 and 4 in Itskhoki and Mukhin (2021); the Belgian moments in the seventh column are from Section 2. Each cell in the third, fourth, fifth, and sixth columns of the table is the median value of moments across 10,000 simulations of 120 quarters; I choose $\kappa$, $\sigma_a$, and $\rho_{a,a^*}$ to respectively match the targeted moments $std(\Delta z_t)/std(\Delta y_t) = 2.5$, $corr(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.4$, and $corr(\Delta y_t, \Delta y_t^*) = 0.35$. In the calibration in the upper part of the table, I set $\kappa = 6.8$, $\sigma_a = 2.5$, and $\rho_{a,a^*} = 0.37$ to match the targeted moments in the model of Itskhoki and Mukhin (2021) with $\gamma = 0.07$. In the calibration in the middle part, I set $\kappa = 5$, $\sigma_a = 5$, and $\rho_{a,a^*} = 0.45$ to match the targeted moments in the model of Itskhoki and Mukhin (2021) with $\gamma = 0.25$. In the calibration in the lower part, I set $\kappa = 9$, $\sigma_a = 2.9$, and $\rho_{a,a^*} = 0.45$ to match the targeted moments in my model with exporter-importer firms with $\gamma = 0.25$ and no price and wage stickiness; the last row of the table indicates whether the model is calibrated when taking price and wage stickiness à la Calvo (1983) into account.

Table 5: Net-Exports Decomposition without and with Exporter-Importer Firms

| Floating Regime ($\omega_e = 0$) | US Moments | No Exporter-Importer Firms ($\phi^e = 0$) | Exporter-Importer Firms ($\phi^e = 0.74$) |
|---|---|---|---|
| | | United States | |
| | | $\gamma = 0.07$ | |
| $\sigma(\Delta e_t)/\sigma(\Delta y_t)$ | 5.2 | 3.38 | 3.86 |
| $\sigma(\Delta q_t)/\sigma(\Delta exports_t)$ | 5.4 | 0.93 | 2.99 |
| $\sigma(\Delta q_t)/\sigma(\Delta imports_t)$ | 5.4 | 0.94 | 3.10 |
| $\sigma(\Delta nx_t)/\sigma(\Delta q_t)$ | 0.10 | 0.17 | 0.09 |
| Price and Wage Stickiness | | YES | YES |

*Notes:* The first and the fourth US moments in the second column are respectively from Tables 3 and 4 in Itskhoki and Mukhin (2021); the second and the third US moments in the second column are from Section 2. $exports_t = y^*_{Ht}$ without exporter-importer firms and $exports_t = y^*_{Ht} + e_{Ht}$ with exporter-importer firms. $imports_t = y_{Ft}$ without exporter-importer firms and $imports_t = y_{Ft} + e^*_{Ft}$ with exporter-importer firms. Each cell in the third and fourth columns of the table is the median value of moments across 10,000 simulations of 120 quarters; I choose $\kappa$, $\sigma_a$, and $\rho_{a,a^*}$ to respectively match the targeted moments $std(\Delta z_t)/std(\Delta y_t) = 2.5$, $corr(\Delta q_t, \Delta c_t - \Delta c^*_t) = -0.4$, and $corr(\Delta y_t, \Delta y^*_t) = 0.35$: I set $\kappa = 6.8$, $\sigma_a = 2.5$, and $\rho_{a,a^*} = 0.37$ to match the targeted moments in the model of Itskhoki and Mukhin (2021) with $\gamma = 0.07$. The last row of the table indicates whether the model is calibrated when taking price and wage stickiness à la Calvo (1983) into account.

Table 6: Quantitative Results for Belgium across Exchange-Rate Regimes

| Pegged Regime | $\omega_e = 0.23$ | Belgian Moments |
|---|---|---|
| $corr(\Delta e_t, \Delta q_t)$ | 0.99 | 0.61 |
| $\sigma(\Delta e_t)/\sigma(\Delta y_t)$ | 1.73 | 1.10 |
| $\sigma(\Delta e_t)/\sigma(\Delta c_t)$ | 2.96 | 1.39 |
| $\sigma(\Delta nx_t)/\sigma(\Delta q_t)$ | 0.28 | 0.88 |
| **Floating Regime** | $\omega_e = 0$ | **Belgian Moments** |
| $corr(\Delta e_t, \Delta q_t)$ | 0.98 | 0.95 |
| $\sigma(\Delta e_t)/\sigma(\Delta y_t)$ | 2.32 | 3.74 |
| $\sigma(\Delta e_t)/\sigma(\Delta c_t)$ | 4.85 | 4.79 |
| $\sigma(\Delta nx_t)/\sigma(\Delta q_t)$ | 0.28 | 0.17 |

*Notes:* The Belgian moments in the third column are from Section 2. Each cell in the second column is the median value of moments across 10,000 simulations of 120 quarters; I choose $\kappa = 9$, $\sigma_a = 2.9$, and $\rho_{a,a^*} = 0.45$ to respectively match the targeted moments $std(\Delta z_t)/std(\Delta y_t) = 2.5$, $corr(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.4$, and $corr(\Delta y_t, \Delta y_t^*) = 0.35$ under the floating regime; I set $\omega_e = 0.23$, as in Itskhoki and Mukhin (2022), under the pegged regime.

# 4 Conclusion

How should researchers think about exchange-rate regimes? In this paper, consistently with the previous literature on the Mussa puzzle and exchange rate disconnect, I show that such regimes affect the volatilities of nominal and real exchange rates but not the volatilities of other real macroeconomic variables, even for economies that have larger exports and imports, compared to total output, than the United States. I also provide a set of assumptions under which modeling this muted reaction is possible, and I show how this result crucially relies on exporters also being firms that intensively import. In the future, I plan to investigate three further questions.

First, is the import intensity of the exporters a structural parameter of the economy—as I assume in my model—or is it endogenous to the exchange-rate regime? In other words, does $\phi^e$ adjust at the time of an exchange-rate regime break that modifies the volatility of the nominal exchange rate? The question has to be systematically investigated at the micro level by asking: how do firms adjust their production function immediately before and after an exchange-rate regime break that changes the volatility of the nominal exchange rate? The model developed here could be easily extended to account for this additional feature of firm optimization.

Second, using the results from a simple generalized autoregressive conditional heteroskedasticity model on the nominal– and real–exchange-rate series, we can empirically observe that during episodes of very high inflation and hyperinflation, the real exchange rate comoves less and less with the nominal exchange rate, resulting in a weakening of the Mussa puzzle. In such cases, exchange-rate regimes interact with inflationary regimes, an empirical result that is at odds with the price stability that ultimately generates the Mussa puzzle. However, this can be modeled in light of menu-cost pricing à la Mankiw (1985): if exogenous shocks affect nominal exchange rates when firms are already changing their prices because of the inflationary regime, such shocks can be incorporated in the new prices at no additional cost.

Third, I emphasize that the exogenous shock in the exchange-rate regime appears in the theoretical model, and its calibration, as a different value of parameter $\omega_e$. This is not necessarily true in the case of a regime break that endogenously arises in response to conditions that are exogenous to the two economies.

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# Appendix for

# "The Macro Neutrality of Exchange-Rate Regimes in the presence of Exporter-Importer Firms"

## A.2 - Empirical Facts and Exchange-Rate Regimes

Table 7: Available Time Periods for the Macroeconomic Variables

| Country | Exchange Rates | Real Macroeconomic Variables |
|---|---|---|
| Australia[a] | 1/1957-12/2019 | 1/1960-12/2019 |
| Austria | 1/1957-12/2019 | 1/1960-12/2019 |
| Belgium | 1/1957-12/2019 | 1/1960-12/2019 |
| Brazil | 12/1979-12/2019 | 1/1996-12/2019 |
| Canada | 1/1957-12/2019 | 1/1961-12/2019 |
| Czech Republic | 3/1993-12/2019 | 1/1994-12/2019 |
| Denmark[b] | 1/1957-12/2019 | 1/1960-12/2019 |
| Estonia | 2/1993-12/2019 | 1/1995-12/2019 |
| Finland[c] | 1/1957-12/2019 | 1/1960-12/2019 |
| France[d] | 1/1957-12/2019 | 1/1960-12/2019 |

| Country | Exchange Rates | Real Macroeconomic Variables |
|---|---|---|
| Germany | 1/1957-12/2019 | 1/1960-12/2019 |
| Greece | 1/1957-12/2019 | 1/1960-12/2019 |
| Ireland[e] | 1/1957-12/2019 | 1/1960-12/2019 |
| Italy[d] | 1/1957-12/2019 | 1/1960-12/2019 |
| Japan | 1/1957-12/2019 | 1/1960-12/2019 |
| Latvia | 6/1993-12/2019 | 1/1995-12/2019 |
| Lithuania | 9/1993-12/2019 | 1/1995-12/2019 |
| Luxembourg | 1/1957-12/2019 | 1/1960-12/2019 |
| Netherlands | 1/1957-12/2019 | 1/1960-12/2019 |
| Norway | 1/1957-12/2019 | 1/1960-12/2019 |

| Country | Exchange Rates | Real Macroeconomics Variables |
|---|---|---|
| Poland[f] | 1/1988-12/2019 | 1/1995-12/2019 |
| Portugal[d] | 1/1957-12/2019 | 1/1960-12/2019 |
| Slovak Republic | 3/1993-12/2019 | 1/1993-12/2019 |
| Slovenia[g] | 3/1992-12/2019 | 1/1995-12/2019 |
| South Africa | 1/1964-12/2019 | 1/1960-12/2019 |
| Spain[h] | 1/1957-12/2019 | 1/1960-12/2019 |
| Sweden | 1/1957-12/2019 | 1/1960-12/2019 |
| Switzerland | 1/1957-12/2019 | 1/1960-12/2019 |
| United Kingdom | 1/1957-12/2019 | 1/1957-12/2019 |
| United States | 1/1957-12/2019 | 1/1957-12/2019 |

*Notes:* [a]The monthly consumer price index is missing, I construct it by linear interpolation using the quarterly consumer price index. [b]The monthly consumer price index from January 1957 to December 1966 is missing, I construct it by linear interpolation using the quarterly consumer price index. [c]Given that the Lavielle (1999) and Lavielle and Moulines (2000) test is particularly sensitive to observations in the series that significantly depart from the rest, I run it only over the period from January 1963 to December 2019. [d]Given that the Lavielle (1999) and Lavielle and Moulines (2000) test is particularly sensitive to observations in the series that significantly depart from the rest, I run it only over the period from January 1959 to December 2019. [e]The monthly consumer price index from January 1957 to December 1996 is missing, I construct it by linear interpolation using the quarterly consumer price index. [f]Given that the Lavielle (1999) and Lavielle and Moulines (2000) test is particularly sensitive to observations in the series that significantly depart from the rest, I run it only over the period from January 1990 to December 2019. [g]Given that the Lavielle (1999) and Lavielle and Moulines (2000) test is particularly sensitive to observations in the series that significantly depart from the rest, I run it only over the period from March 1992 to December 2006. [h]Given that the Lavielle (1999) and Lavielle and Moulines (2000) test is particularly sensitive to observations in the series that significantly depart from the rest, I run it only over the period from January 1960 to December 2019; the bilateral nominal exchange rate in March 1964 is missing, I construct it by linear interpolation.

### A.2.1 - A Characterization of Exchange-Rate Regimes

I apply the test developed by Lavielle (1999) and Lavielle and Moulines (2000) to empirically identify structural breaks in the volatility of the nominal- and real-exchange-rate series. It is an extension of the Bai and Perron (1998) test for weakly and strongly dependent processes and is used to simultaneously detect structural breaks in the volatility of a time series when the number of structural breaks is unknown. I preliminarily remove the outliers from the $\Delta e_t$ ($\Delta q_t$) series to properly apply the heteroskedasticity-break test, defining outliers as elements more than three local standard deviations away from the local mean within a forty-nine-month window that is centered about the current element and contains forty-eight neighboring months.

**The Lavielle (1999) and Lavielle and Moulines (2000) test.** Denote $X_t = \Delta e_t$ (or $X_t = \Delta q_t$), $t = 1, 2, 3, ..., T$. Assume that the unknown number of segments $K$ in the time series is upper bounded by a known finite $\bar{K}$. Lavielle (1999) and Lavielle and Moulines (2000) propose to estimate the configuration of structural breaks $\tau$ and the number of segments $K$ by minimizing the penalized-contrast function as follows:

$$(\hat{\tau}_T, \hat{K}_T) = \arg \min_{1 \leq K \leq \bar{K}} \inf_{\tau \in \mathcal{T}_K} \left\{ \frac{1}{T} \sum_{k=1}^{K} \left( \frac{||\mathbf{X}_k||^2}{\sigma_k^2} + T_k \ln \sigma_k^2 \right) + \beta_T K \right\}.$$

Here, $\mathbf{X}_k$ is the vector of observations that belong to segment $k$ in the configuration $\tau = (\tau_k, 1 \leq k \leq K-1)$, $T_k$ is the length of $\mathbf{X}_k$, $\sigma_k^2$ is the variance of $X_t$ in segment $k$, and $\beta_T K$ is the penalization term. In my analysis, I set $\bar{K} = 6$, which implies a maximum of five structural breaks, and I choose $\beta_T$ according to Lavielle (1999, p. 81).

Table 8: Exchange-Rate Regimes

| Country | Nominal Exchange Rate | Real Exchange Rate | Exchange-Rate Regime |
|---|---|---|---|
| Australia | January 1957 - December 1970 | January 1957 - November 1972 | Pegged Regime |
| | January 1971 - December 2019 | December 1972 - December 2019 | Floating Regime |
| Austria | January 1957 - November 1970 | January 1957 - December 1972 | Pegged Regime |
| | December 1970 - October 1979 | January 1973 - March 1979 | Floating Regime |
| | November 1979 - July 1992 | April 1979 - July 1992 | Pegged Regime |
| | August 1992 - August 1998 | August 1992 - March 1997 | Floating Regime |
| | September 1998 - December 2019 | April 1997 - December 2019 | Pegged Regime |
| Brazil | December 1979 - August 1994 | December 1979 - November 1994 | Floating Regime |
| | September 1994 - July 1998 | December 1994 - November 1996 | Pegged Regime |
| | August 1998 - December 2019 | December 1996 - December 2019 | Floating Regime |
| Canada | January 1957 - April 1970 | January 1957 - December 1972 | Pegged Regime |
| | May 1970 - December 2019 | January 1973 - December 2019 | Floating Regime |
| Czech Republic | March 1993 - December 2012 | March 1993 - October 2012 | Floating Regime |
| | January 2013 - December 2019 | November 2012 - December 2019 | Pegged Regime |
| Denmark | January 1957 - December 1972 | January 1957 - November 1972 | Pegged Regime |
| | January 1973 - December 1978 | December 1972 - December 1977 | Floating Regime |
| | January 1979 - July 1992 | January 1978 - July 1992 | Pegged Regime |
| | August 1992 - September 1997 | August 1992 - February 1997 | Floating Regime |
| | October 1997 - December 2019 | March 1997 - December 2019 | Pegged Regime |

| Country | Nominal Exchange Rate | Real Exchange Rate | Exchange-Rate Regime |
|---|---|---|---|
| Estonia | February 1993 - March 1998 | February 1993 - April 1997 | Floating Regime |
| | April 1998 - December 2019 | May 1997 - December 2019 | Pegged Regime |
| Finland | January 1957 - June 1971 | January 1957 - April 1971 | Pegged Regime |
| | July 1971 - March 1983 | May 1971 - May 1983 | Floating Regime |
| | April 1983 - September 1991 | June 1983 - October 1991 | Pegged Regime |
| | October 1991 - July 1997 | November 1991 - April 1997 | Floating Regime |
| | August 1997 - December 2019 | May 1997 - December 2019 | Pegged Regime |
| France | January 1957 - July 1969 | January 1957 - July 1971 | Pegged Regime |
| | August 1969 - March 1978 | August 1971 - March 1983 | Floating Regime |
| | April 1978 - July 1992 | April 1983 - July 1992 | Pegged Regime |
| | August 1992 - August 1998 | August 1992 - July 1997 | Floating Regime |
| | September 1998 - December 2019 | August 1997 - December 2019 | Pegged Regime |
| Germany | January 1957 - November 1970 | January 1957 - December 1972 | Pegged Regime |
| | December 1970 - September 1978 | January 1973 - September 1981 | Floating Regime |
| | October 1978 - July 1992 | October 1981 - July 1992 | Pegged Regime |
| | August 1992 - March 1998 | August 1992 - March 1997 | Floating Regime |
| | April 1998 - December 2019 | April 1997 - December 2019 | Pegged Regime |

| Country | Nominal Exchange Rate | Real Exchange Rate | Exchange-Rate Regime |
|---|---|---|---|
| Greece | January 1957 - July 1970 | January 1957 - December 1970 | Pegged Regime |
| | August 1970 - October 2000 | January 1971 - July 1995 | Floating Regime |
| | November 2000 - December 2019 | August 1995 - December 2019 | Pegged Regime |
| Ireland | January 1957 - March 1970 | January 1957 - August 1972 | Pegged Regime |
| | April 1970 - September 1981 | September 1972 - February 1982 | Floating Regime |
| | October 1981 - July 1992 | March 1982 - July 1992 | Pegged Regime |
| | August 1992 - October 1998 | August 1992 - March 1998 | Floating Regime |
| | November 1998 - December 2019 | April 1998 - December 2019 | Pegged Regime |
| Italy | January 1957 - December 1972 | January 1957 - January 1975 | Pegged Regime |
| | January 1973 - April 1981 | February 1975 - July 1983 | Floating Regime |
| | May 1981 - May 1992 | August 1983 - May 1992 | Pegged Regime |
| | June 1992 - December 1998 | June 1992 - May 1996 | Floating Regime |
| | January 1999 - December 2019 | June 1996 - December 2019 | Pegged Regime |
| Japan | January 1957 - July 1971 | January 1957 - January 1973 | Pegged Regime |
| | August 1971 - December 2019 | February 1973 - December 2019 | Floating Regime |
| Latvia | June 1993 - December 2004 | June 1993 - March 2004 | Floating Regime |
| | January 2005 - December 2019 | April 2004 - December 2019 | Pegged Regime |
| Lithuania | September 1993 - January 2002 | September 1993 - February 2002 | Floating Regime |
| | February 2002 - December 2019 | March 2002 - December 2019 | Pegged Regime |

| Country | Nominal Exchange Rate | Real Exchange Rate | Exchange-Rate Regime |
|---|---|---|---|
| Luxembourg | January 1957 - June 1971 | January 1957 - December 1972 | Pegged Regime |
| | July 1971 - September 1978 | January 1973 - January 1980 | Floating Regime |
| | October 1978 - July 1992 | February 1980 - July 1992 | Pegged Regime |
| | August 1992 - March 1998 | August 1992 - April 1996 | Floating Regime |
| | April 1998 - December 2019 | May 1996 - December 2019 | Pegged Regime |
| Netherlands | January 1957 - March 1971 | January 1957 - December 1972 | Pegged Regime |
| | April 1971 - December 1978 | January 1973 - May 1978 | Floating Regime |
| | January 1979 - July 1992 | June 1978 - July 1992 | Pegged Regime |
| | August 1992 - March 1998 | August 1992 - May 1997 | Floating Regime |
| | April 1998 - December 2019 | June 1997 - December 2019 | Pegged Regime |
| Norway | January 1957 - June 1971 | January 1957 - December 1972 | Pegged Regime |
| | July 1971 - December 2019 | January 1973 - December 2019 | Floating Regime |
| Poland | January 1988 - July 2012 | January 1988 - April 2012 | Floating Regime |
| | August 2012 - December 2019 | May 2012 - December 2019 | Pegged Regime |
| Portugal | January 1957 - June 1971 | January 1957 - July 1971 | Pegged Regime |
| | July 1971 - September 1986 | August 1971 - October 1975 | Floating Regime |
| | October 1986 - May 1991 | November 1975 - May 1978 | Pegged Regime |
| | June 1991 - March 1998 | June 1978 - June 1997 | Floating Regime |
| | April 1998 - December 2019 | July 1997 - December 2019 | Pegged Regime |

| Country | Nominal Exchange Rate | Real Exchange Rate | Exchange-Rate Regime |
|---|---|---|---|
| Slovak Republic | March 1993 - December 2008 | March 1993 - December 2008 | Floating Regime |
| | January 2009 - December 2019 | January 2009 - December 2019 | Pegged Regime |
| Slovenia | March 1992 - July 1997 | March 1992 - May 1998 | Floating Regime |
| | August 1997 - December 2019 | June 1998 - December 2019 | Pegged Regime |
| South Africa | January 1964 - November 1970 | January 1964 - February 1970 | Pegged Regime |
| | December 1970 - December 2019 | March 1970 - December 2019 | Floating Regime |
| Spain | January 1957 - September 1968 | January 1957 - January 1971 | Pegged Regime |
| | October 1968 - April 1998 | February 1971 - May 1996 | Floating Regime |
| | May 1998 - December 2019 | June 1996 - December 2019 | Pegged Regime |
| Sweden | January 1957 - February 1973 | January 1957 - February 1973 | Pegged Regime |
| | March 1973 - December 2019 | March 1973 - December 2019 | Floating Regime |
| Switzerland | January 1957 - August 1970 | January 1957 - November 1972 | Pegged Regime |
| | September 1970 - December 2019 | December 1972 - December 2019 | Floating Regime |
| United Kingdom | January 1957 - November 1970 | January 1957 - August 1972 | Pegged Regime |
| | December 1970 - December 2019 | September 1972 - December 2019 | Floating Regime |
| United States | January 1957 - June 1971 | January 1957 - July 1971 | Pegged Regime |
| | July 1971 - December 2019 | August 1971 - December 2019 | Floating Regime |

# A.3 - Theoretical Framework

## A.3.1 - Model

### A.3.1.1 - Home Country

**Households.** The solution to the consumption-savings problem of the representative household can be obtained by formulating a Lagrangian. Combining the first-order conditions of the Lagrangian and the accumulation rule for the capital stock results in the Euler equation, the labor supply equation, and the asset pricing equation:

$$C_t^{-\sigma} = \beta R_t \mathbb{E}_t \left[ \frac{C_{t+1}^{-\sigma}}{P_{t+1}} \right] P_t, \tag{8}$$

$$C_t^{\sigma} L_t^{1/\varphi} = \frac{W_t}{P_t}, \text{ and} \tag{9}$$

$$\left(1 + \kappa \frac{\Delta K_{t+1}}{K_t}\right) C_t^{-\sigma} = \beta \mathbb{E}_t \left\{ \left(C_{t+1}^{-\sigma}\right) \left[ \frac{R_{t+1}^K}{P_{t+1}} - \delta + \left(1 + \kappa \frac{\Delta K_{t+2}}{K_{t+1}}\right) + \frac{\kappa}{2} \frac{\left(\kappa \frac{\Delta K_{t+2}}{K_{t+1}}\right)^2}{2\kappa} \right] \right\}. \tag{10}$$

**Domestic firms.** The solution to the cost minimization of the representative domestic firm results in the following demands for labor, capital, and intermediate inputs:

$$W_t L_{Dt} = (1 - \phi)(1 - \vartheta) MC_{Dt} Y_{Ht}, \tag{11}$$

$$R_t^K K_{Dt} = (1 - \phi) \vartheta MC_{Dt} Y_{Ht}, \text{ and} \tag{12}$$

$$P_t X_{Dt} = \phi MC_{Dt} Y_{Ht}. \tag{13}$$

**Exporter-importer firms.** The solution to the cost minimization of the representative exporter-importer firm results in the following demands for labor, capital, intermediate inputs, and directly imported inputs:

$$W_t L_{Et} = (1 - \phi^e)(1 - \phi)(1 - \vartheta) MC_{Et} Y_{Ht}^*, \tag{14}$$

$$R_t^K K_{Et} = (1 - \phi^e)(1 - \phi) \vartheta MC_{Et} Y_{Ht}^*, \tag{15}$$

$$P_t X_{Et} = (1-\phi^e)\phi MC_{Et} Y^*_{Ht}, \text{ and} \tag{16}$$

$$\mathcal{E}_t P^*_{Ft} E^*_{Ft} = \phi^e MC_{Et} Y^*_{Ht}. \tag{17}$$

#### A.3.1.5 - Equilibrium Definition and Model Solution

**Equilibrium in the Nonlinear Model.** Given the exogenous shocks $\{a_t, a^*_t\}$, the policy specifications for the sequence of gross nominal interest rates $\{R_t, R^*_t\}$, and the targeted nominal exchange rate $\{\bar{\mathcal{E}}\}$, an equilibrium in the nonlinear model is a collection of stochastic processes for $\{Y_t, Y^*_t, Y_{Ht}, Y_{Ft}, Y^*_{Ht}, Y^*_{Ft}, C_t, C^*_t, L_t, L^*_t, K_t, K^*_t, X_t, X^*_t, P_t, P^*_t, \mathcal{P}_t, \mathcal{P}^*_t, P_{Ht}, P_{Ft}, P^*_{Ht}, P^*_{Ft}, W_t, W^*_t, R^K_t, R^{K^*}_t, Q_t, Q^*_t, B_t, B^*_t, \mathcal{E}_t\}$ that solves the price indexes (1) and (2); the optimal pricing equations (3) and (4); the Euler equation (8); the labor supply equation (9); the asset pricing equation (10); the domestic firm demands for labor (11), capital (12), and intermediate inputs (13); the exporter-importer firm demands for labor (14), capital (15), intermediate inputs (16), and directly imported inputs (17); their respective counterparts in the foreign country; the market clearing conditions for labor, capital, and goods in the home country; the market clearing conditions for labor, capital, and goods in the foreign country; and the market clearing conditions in the international financial markets.

### A.3.2 - Exporter-Importer Firms Resolving Exchange Rate Disconnect

**Proof.**[30] The labor supply (9) and labor demand (11, 14) equations are:

$$\sigma c_t + \varphi^{-1} l_t = w_t - p_t, \text{ and}$$

$$w_t + l_t = (1-\tau)(mc_t + y_{Ht}) + \tau(mc_{Et} + y^*_{Ht}).$$

Here, $\tau \equiv (1-\phi^e)\gamma$. Combining the two to solve for $l_t$, and using the expressions for the marginal costs and the optimal pricing equations, results in:

$$\varphi\sigma c_t + [(1-\tau)y_{Ht} + \tau y^*_{Ht}] = (1+\varphi)a_t - \left[\frac{\varphi+\phi}{1-\phi}\iota + \tau\phi^e(1+2\iota-\alpha\iota)\right] q_t.$$

[30] I prove the result in the model with no capital in the same fashion of Itskhoki and Mukhin (2021, 2022).

Here, $\iota \equiv \frac{\tilde{\gamma}(1-\delta)(1-\alpha)(1-\tilde{\gamma})+[\tilde{\gamma}(1-\delta)]^2}{[(1-\alpha)(1-\tilde{\gamma})]^2-[\tilde{\gamma}(1-\delta)]^2}$ and $\delta \equiv \frac{\alpha(1-\tilde{\gamma})+(1-\alpha)\phi^e}{1-\tilde{\gamma}[1-(1-\alpha)\phi^e]}$. Subtracting the symmetric equation for the foreign country yields to the following equation that characterizes the supply side:

$$\varphi\sigma(c_t - c_t^*) + [(1-\tau)(y_{Ht} - y_{Ft}^*) + \tau(y_{Ht}^* - y_{Ft})] = (1+\varphi)(a_t - a_t^*) - 2\left[\frac{\varphi+\phi}{1-\phi}\iota + \tau\phi^e(1+2\iota-\alpha\iota)\right] q_t. \quad (18)$$

Combining the demands in each good market, and using the demands for intermediate inputs and directly imported intermediate inputs, yields to the following equation that characterizes the demand side:

$$[(1-\tau)(y_{Ht} - y_{Ft}^*) + \tau(y_{Ht}^* - y_{Ft})] = \frac{(1-\phi)(1-2\tau)}{1+\phi(2\tau-1)}(c_t - c_t^*) + \frac{2\zeta}{1+\phi(2\tau-1)} q_t. \quad (19)$$

Here,

$$\begin{aligned}\zeta \equiv & (1-\tau)\frac{1-\gamma}{(1-\gamma)+\phi^e\gamma}\left\{\theta(1-\alpha)\iota + \phi\left[\phi^e\tau(1+2\iota-\alpha\iota) - \iota\right]\right\} + \\ & (1-\tau)\frac{\phi^e\gamma}{(1-\gamma)+\phi^e\gamma}\left\{\left[1-\phi^e(1-\phi\tau)\right](1+2\iota-\alpha\iota) - \theta(1-\delta)(1+\iota) - \phi\iota\right\} + \\ & \tau\left\{\phi\left[\phi^e\tau(1+2\iota-\alpha\iota) - \iota\right] - \theta(1-\delta)(1+\iota)\right\}.\end{aligned}$$

Combining equations (18) and (19) to solve for $c_t - c_t^*$ results in:

$$\begin{aligned}c_t - c_t^* = & \frac{(1+\varphi)\left[1+\phi(2\tau-1)\right]}{\varphi\sigma\left[1+\phi(2\tau-1)\right] + (1-\phi)(1-2\tau)}(a_t - a_t^*) + \\ & - 2\frac{1+\phi(2\tau-1)}{\varphi\sigma\left[1+\phi(2\tau-1)\right] + (1-\phi)(1-2\tau)}\left[\frac{\varphi+\phi}{1-\phi}\iota + \tau\phi^e(1+2\iota-\alpha\iota) + \frac{\zeta}{1+\phi(2\tau-1)}\right] q_t.\end{aligned}$$

Then, $c_t - c_t^* = \frac{(1+\varphi)}{1+\varphi\sigma}(a_t - a_t^*)$ if $\phi^e \to 1$.

□

Table 9: Model Parameters

| Non-Calibrated Parameter | Variable | Value | Source |
|---|---|---|---|
| Household discount factor | $\beta$ | 0.99 | |
| Relative risk aversion | $\sigma$ | 2 | |
| Inverse Frisch elasticity | $\varphi$ | 1 | |
| Intermediate inputs share | $\phi$ | 0.5 | |
| Capital share | $\vartheta$ | 0.3 | |
| Depreciation rate | $\delta$ | 0.02 | |
| Exporters' import intensity | $\phi^e$ | 0.74 | Amiti, Itskhoki, and Konings (2014) |
| International-Trade openness for the United States | $\gamma$ | 0.07 | |
| International-Trade openness for Belgium | $\gamma$ | 0.25 | Belgian Imports-to-GDP ratio = 0.50 |
| Elasticity of substitution | $\theta$ | 1.5 | |
| Strategic complementarity | $\alpha$ | 0.4 | |
| Interest rate smoothing | $\rho$ | 0.95 | |
| Inflation rate targeting | $\omega_\pi$ | 2.15 | |
| Nominal exchange rate targeting, under pegged regimes | $\omega_e$ | 0.23 | Itskhoki and Mukhin (2022) |
| Persistence of the shocks | $\rho_a$, $\rho_{a^*}$, $\rho_\psi$ | 0.97 | |
| Standard deviation of financial shocks | $\chi_1\sigma_\psi$ | 1 | |
| Net Foreign Asset Coefficient in equation (5) | $\chi_2$ | 0.001 | |
| Calvo probability for prices | $\lambda_p$ | 0.75 | |
| Calvo probability for wages | $\lambda_w$ | 0.85 | |

| Calibrated Parameter | Variable | | Targeted Moment |
|---|---|---|---|
| Capital adjustment cost | $\kappa$ | $\rightarrow$ | $std(\Delta z_t)/std(\Delta y_t) = 2.5$ |
| Standard deviation of productivity shocks | $\sigma_a$, $\sigma_{a^*}$ | $\rightarrow$ | $corr(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.4$ |
| Correlation of productivity shocks | $\rho_{a,a^*}$ | $\rightarrow$ | $corr(\Delta y_t, \Delta y_t^*) = 0.35$ |

*Notes:* The non-calibrated parameters are taken from Itskhoki and Mukhin (2021) if not differently specified; for details on the values of the calibrated parameters see Tables 4, 5, and 6 in Section 3.3: I choose $\kappa$, $\sigma_a$, and $\rho_{a,a^*}$ to respectively match the targeted moments $std(\Delta z_t)/std(\Delta y_t) = 2.5$, $corr(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.4$, and $corr(\Delta y_t, \Delta y_t^*) = 0.35$ under the floating regime.