

Mr. Keynes and the “Classics”; A Suggested Reconciliation

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Abstract

This paper proposes resolving the controversy between the Keynesians and the monetarists (“classics”). The dispute dates to Keynes’s book *The General Theory of Employment, Interest and Money* (1936)—famously formalized in Hicks’s article “Mr. Keynes and the “Classics”; A Suggested Interpretation” (1937)—and became the subject of several studies even if never reaching a definite conclusion. We first overturn existing empirical findings, relating money demand and long-term interest rates, using more recent data. We then reconcile the Keynesian and monetarist perspectives theoretically and empirically, using New-Keynesian general-equilibrium theory and different assumptions about the underlying monetary-policy regime.

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1 Introduction

This paper proposes a resolution to one of the most famous controversies in macroeconomics, the one between the Keynesians and the monetarists (“classics”). Keynesians maintained that increasing the money supply beyond a certain point did not affect macroeconomic aggregates. In contrast, the “classics,” and later the monetarists, maintained that the monetary authority can always stimulate the economy by increasing the money supply. We resolve the controversy, using New-Keynesian general-equilibrium theory, and reconcile the two schools of thought. Our resolution is ultimately startling and favors neither school of thought: the predictions of either camp can be supported by relying on different *monetary-policy regimes*, where our definition of policy regime follows Sargent (1982).

Economists have well understood that a higher money supply can boost spending by increasing inflation expectations and reducing real interest rates, at least since Romer (1992), even when the zero lower bound (ZLB) may be binding. Krugman (1998) is the first to demonstrate in a modern general-equilibrium model that *permanent* money supply expansions increase aggregate demand via expected inflation, generating lower real interest rates. In contrast, if the central bank increases the money supply only temporary it has no effect on macroeconomic variables such as interest rates, output, and inflation. Following this work, the role of expected future money supply and interest policy, together with the modern treatment of expectations, is one of the major themes of the literature—see, for instance, Benhabib, Schmitt-Grohé, and Uribe (2002), Eggertsson and Woodford (2003), Svensson (2000, 2003), Auerbach and Obstfeld (2005), Woodford (2012), and Galí (2020).

The main contribution of our paper is to characterize policy regimes that rationalize two schools of thought—Keynesian and monetarist—which still cast a long shadow on today’s economic debates, applying insights from the modern literature on liquidity traps and monetary policy effectiveness. By doing so, we resolve the long-standing controversy between these schools theoretically and empirically.

In 1937, Hicks’s paper “Mr. Keynes and the “Classics”; A Suggested Interpretation” translated Keynes’s *General Theory of Employment, Interest and Money* into a set of equations representing a theoretical framework, which became known as the IS-LM model. Yet, there is another contribution of Hicks’s paper that is not as commonly appreciated but is highlighted in the title of his paper, which we have claimed as our own, replacing “Interpreted” with “Reconciliation.” As evident by the title and as becomes even more apparent on closer reading, Hicks’s paper is not just a mathematical formulation of Keynes’s *General Theory*.

Hicks’s central claim is that, contrary to Keynes’s assertion, his theory is in fact *not* general but a special case that should be considered as the polar opposite of what Hicks termed the “classics”—a label he took to represent the conventional wisdom prior to Keynes. Hicks’s argument is that the world can evolve according to either Keynes or the “classics.” What separated Keynes and the “classics” was not the assumption of price or wage rigidity, which later became the bone of contention in the literature after the Great Inflation of the 1970s.

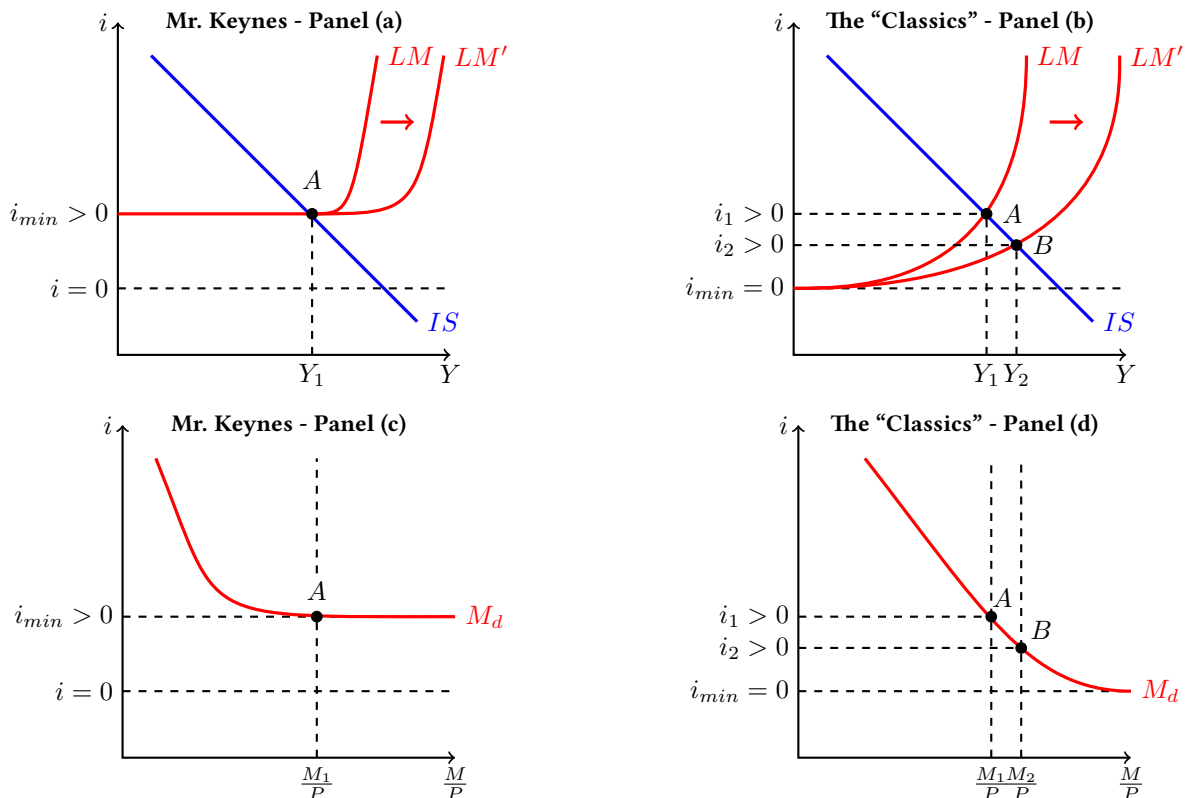


Figure 1: Hicks's Suggested Interpretation

Instead, Hicks argued it was the notion of a liquidity trap: at a certain level of interest rates, Keynes asserted, printing money has no effect on interest rates or output (see Panels [a] and [c] of Figure 1). Meanwhile, the "classics" asserted that there is no such point (see Panels [b] and [d] of Figure 1): increasing money supply always lowers interest rates and thus increases output. Thus, to a modern reader, determining whether the equilibrium is the one prescribed by Keynes or the "classics" boils down to answering a simple question: is the short-term nominal interest rate close to zero or not? How can the answer to such a question, readily inferred from observable data, have resulted in a controversy which stretched over decades? Here, we come to the matter which goes to the core of the controversy.

Both Keynes and the "classics," and later the Keynesians and the monetarists—but the monetarists adopted essentially the same position Hicks attributed to the "classics"—*assumed* that the demand for money depends on the long-term nominal interest rate instead of the short-term nominal interest rate. The controversy, then, could be simply resolved by estimating the elasticity of money demand with respect to the long-term interest rate by formal statistical tests.¹

¹The literature has recognized for a long time that the relevant opportunity cost of holding money is the short-term interest rate. For instance, Lucas (1988) makes this explicit in a Carnegie Rochester Volume article written to celebrate Allan Meltzer's contribution to monetary economics.

We resolve the controversy in three steps. In Section 2, we first survey the literature, whose objective was to test for the existence of a liquidity trap using US data from 1900 to 1958. The methods we replicate are relatively primitive by current standards. Nevertheless, the replication is essential to clarify the controversy’s empirical nature and anticipate the identification discussion in Section 5. The literature reaches a consensus that the data do not justify the notion of a liquidity trap. However, we show that, once we add data from the Great Inflation of the 1970s and the Great Recession, a liquidity trap exists *according to the statistical tests proposed in this literature*.

Figure 2 illustrates the statistical power of more recent data. The top panel of Figure 2 is a scatter diagram of long-term government-bond interest rates plotted against the ratio of the monetary base to national income in 1900–1958. As the top panel reveals, long-term rates fluctuated within a narrow band between 1900 and 1958. They showed little or no tendency to converge to a positive long-term nominal interest rate, as Keynesians predicted. In light of this, it is not surprising that the literature found no evidence for a bound on long-term rates, which several authors we cite in the paper did not estimate statistically different from zero. We confirm this result using the same data from 1900 to 1958.

The bottom panel of Figure 2 represents the data from 1900 to 2019. With the addition of the years from 1959 to 2019, a curve emerges with obvious nonlinearities. In the bottom panel of Figure 2, long-term rates seem to tend to a positive number, estimated to be 2 percent, depending on the details of the specification. Has the controversy thus been resolved in favor of the Keynesians? According to our theory, the answer is no.

The second step, Sections 3 and 4, illustrates a New-Keynesian IS-LM model to demonstrate that the estimated relationship in Section 2, what we call “quasi-money demand,” is not structural in the sense of Lucas (1976). While the structural money demand depends on the short-term interest rate, the quasi-money demand depends on the *long-term interest rate*. Meanwhile, the long-term rate depends on current and *expected* future short-term rates. Since these rates, in turn, depend upon the assumed policy regime, estimating the quasi-money demand is subject to the Lucas critique.

We show two examples of monetary-policy regimes that can rationalize the Keynesians’ vision of the world, on the one hand, and the monetarists’ vision of the world, on the other hand. Assuming a Keynesian policy regime, the model replicates the intersection of the IS and LM curves shown in panel (a) of Figure 1. In contrast, assuming a monetarist policy regime, the model replicates the intersection of the IS and LM curves shown in panel (b) of Figure 1. The difference between the two is that under the Keynesian policy regime, people expect any increase in the money supply to be temporary if the economy is at the ZLB. In contrast, under the monetarist policy regime, people expect any increase in the money supply to be permanent.

Panel (c) of Figure 1 shows the relationship between the money supply and the long-term interest rate under the Keynesian regime. Any increase in the money supply does not affect the long-term interest rate beyond $i_{min} > 0$. The theory suggests that this happens under the condition that the short-term nominal interest rate is at the ZLB.

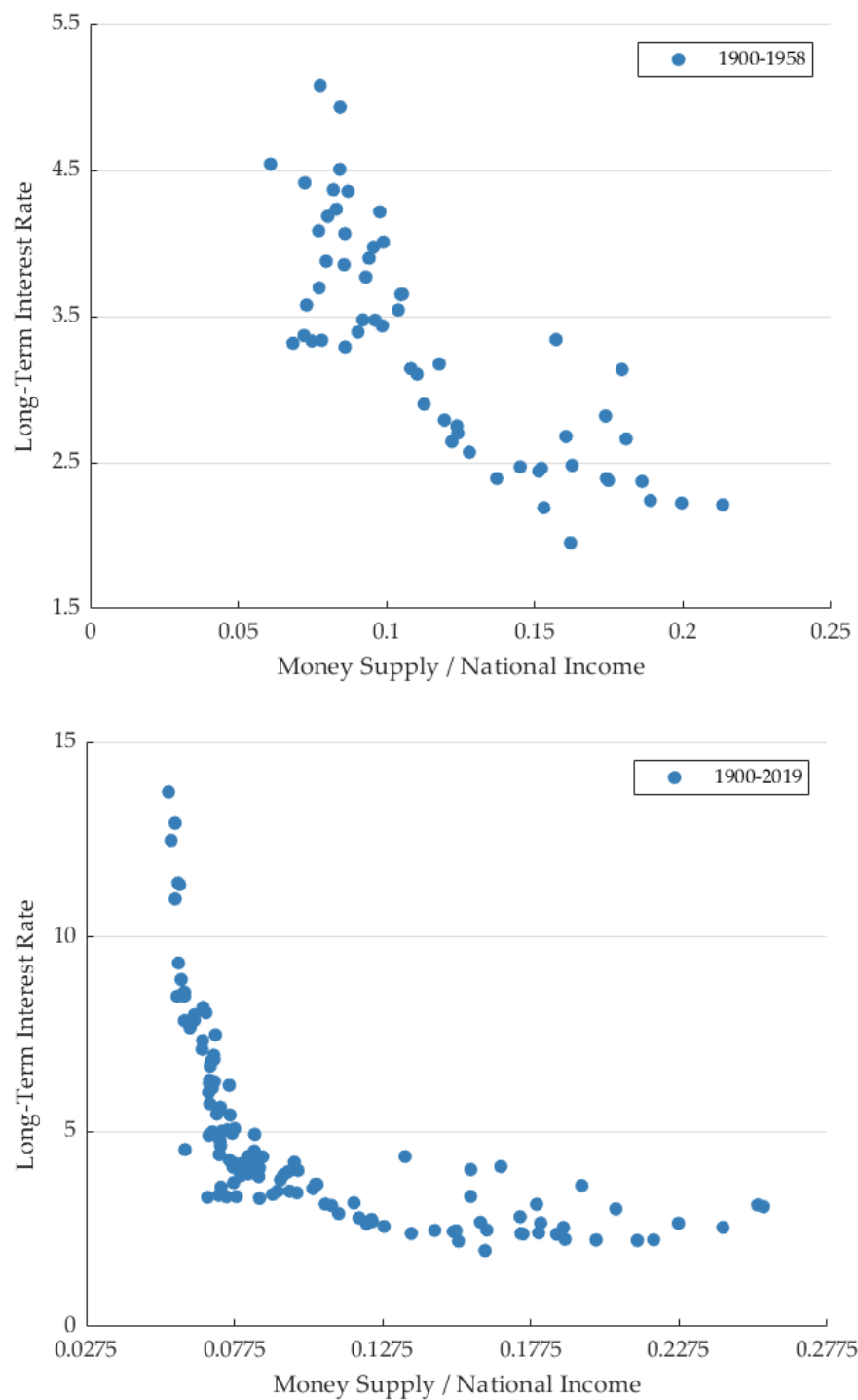


Figure 2: Long-Term Government-Bond Interest Rates versus Money Supply over National Income

In contrast, under the monetarist regime, this relationship takes the form shown in panel (d) of Figure 1: any increase in the money supply continuously decreases the long-term interest rate, even once the short-term interest rate is at the ZLB. The reason is that a permanent increase in the money supply implies a reduction in expected future short-term interest rates generating a reduction in the long-term one, because it depends on current and expected future short-term interest rates.

We take the final step in Section 5, establishing assumptions which allow us to identify the quasi-money demand empirically. We propose an estimation strategy, that follows from the derivation of the model, demonstrating that the quasi-money demand asymptotes at a positive long-term nominal interest rate, which is statistically significant. However, our interpretation of this finding differs from the previous literature since our result does not prove that monetary policy is powerless or “trapped.” It only says that the evidence from the period 1900-2019 implies that long-term nominal interest rates stopped responding to static increases in the money supply, once the ZLB became binding. Thus, it suggests that expectations about future short-term interest rates evolved from 1900 to 2019 *as if* the supply of money was set consistently with the Keynesian policy regime.

Despite this finding, there remains a fundamental role for monetary policy. However, contrary to the monetarists’ contention, the key to successful monetary policy is not static variation in money supply, which feeds into long-term interest nominal rates and aggregate demand via the quasi-money-demand equation. If the ZLB is binding, policymakers can still implement expansionary policy by managing expectations about future actions after the ZLB no longer constrains policy. Managing expectations about future policy, however, requires no open-market operation today or any changes in the money supply.

2 Empirical Estimates for the Liquidity Trap

2.1 Background

The literature testing for the existence of a liquidity trap focused on US data from the period 1900–1958. This literature originated in an article written by Bronfenbrenner and Mayer (1960), who concluded that “neither the data nor theoretical considerations give any reason for expecting a liquidity trap.” Eisner (1963) objected to this article, prompting a rebuttal from Bronfenbrenner and Mayer (1963). Subsequently, Meltzer (1963*b*) published an article concluding that “the evidence lends little or no support to the trap.”

As the literature matured, researchers began employing statistical tests, starting with Pifer (1969) and continuing with White (1972). The overall conclusion, however, remained the same: the data did not provide evidence in favor of a liquidity trap. Below, we replicate this literature and show that more recent data overturns the results.

2.2 Data and Estimation Methods

The strategy the literature converged upon is best explained by considering an illustrative money demand:

$$M_t = \frac{\gamma Y_t^\omega}{(i_t - i_{min})^{-\alpha}} e^{\xi_t}. \quad (1)$$

Here, M_t is money, Y_t is income, i_t is the nominal long-term interest rate, i_{min} is the rate of interest below which long-term interest rates cannot fall, and ξ_t is an exogenous disturbance. The question is whether i_{min} is statistically different from zero. The authors interpreted the result that i_{min} is not statistically different from zero, saying that the data did not support the existence of a liquidity trap.

Following the literature, we present the results using M1 as the measure of money, corporate bonds to measure long-term nominal interest rates, GDP as the measure of income, and an additional measure of total assets. We report all data sources in Table 1 in Appendix A.2.2.1 and consider some alternative measures in Appendix A.2.3.

We replicate Pifer (1969) by formulating the test for whether i_{min} is statistically different from zero, using a two-step maximum likelihood estimation. First, for each given i_{min} , we run the following regression, which we obtain by taking the logarithm of Equation (1). Second, we choose the i_{min} that maximizes the likelihood function:

$$\log(M_t) = \log(\gamma) + \omega \log(Y_t) + \alpha \log(i_t - i_{min}) + \xi_t. \quad (2)$$

We then replicate White (1972) in extending Pifer's method by considering a more general regression:

$$\frac{M_t^\lambda - 1}{\lambda} = \frac{\gamma^\lambda - 1}{\lambda} + \omega \frac{Y_t^\lambda - 1}{\lambda} + \alpha \frac{(i_t - i_{min})^\lambda - 1}{\lambda} + \xi_t. \quad (3)$$

Equation (3) generalizes the specification of Equation (2), which is a special case for $\lambda \rightarrow 0$.²

2.3 Empirical Results

The first column in Table 2 in Appendix A.2.3 shows the results for our replication of Pifer (1969), both with and without total assets as an explanatory variable, focusing on the sample 1900–1958. The estimated i_{min} is 2.24. However, as White (1972) stresses, the result is no longer statistically significant considering the functional form of Equation (3). We replicate White's result in the second and third columns of Table 2 in Appendix A.2.3. The point estimate of i_{min} is 1.70, but the 95 percent confidence interval runs from -0.94 to 2.21. The literature thus concluded that US data showed no evidence of a liquidity trap. The last column of Table 2 in Appendix A.2.3 demonstrates that including more recent data from 1959 to 2019 overturns White's result, as it reduces confidence intervals.

²See Appendix A.2.2.2 for the details of Pifer's estimation; we follow Eisner (1971) to have correct standard errors for the estimate of i_{min} . See Appendix A.2.2.3 for White's estimation details.

The point estimate of i_{min} is 2.23 with a 99 percent confidence interval ranging from 2.05 to 2.30.³ Thus, data from the second half of the twentieth century and the early part of the twenty-first seems to resolve the clash: the Keynesians won. Or did they? Our answer is negative. A more modern analysis shows that the LM curve of the form envisioned by the Keynesians or the monetarists is not structural and can emerge under different policy regimes.

3 A Classic Lucas Critique in a New-Keynesian IS-LM Model

This section presents a New-Keynesian IS-LM model to generate Hicks's diagrams under different assumptions about the monetary-policy regime. We summarize the model with the following three equations, providing detailed micro foundations in Appendix A.3.

The IS equation is

$$\hat{Y}_t = \delta \mathbb{E}_t \hat{Y}_{t+1} - \sigma \delta (\hat{i}_t - \mathbb{E}_t \pi_{t+1} - \hat{r}_t^e), \quad (\text{IS})$$

where $0 < \delta < 1$ is a discounting term and $\sigma > 0$ is the inverse of the relative risk aversion coefficient. All variables are expressed in terms of log deviations from steady state, with \hat{Y}_t denoting output, \hat{i}_t the (gross) short-term risk-free nominal interest rate, \hat{M}_t the money supply, π_t inflation, and \hat{r}_t^e an exogenous disturbance.

The term δ allows us to consider ZLB episodes of arbitrary duration. Various approaches in the literature have justified this term and we provide an example with relative wealth in the utility function in Appendix A.3, following Michaillat and Saez (2021). As stressed by Eggertsson, Mehrotra, and Robbins (2019), we cannot in fact consider a *permanent* liquidity trap in a standard representative-agent model. Yet, the previous literature placed this possibility at the center of the controversy and, therefore, we include the term δ , allowing for a permanent demand recession, to formally characterize our discussion.⁴

Inflation and output satisfy a Phillips curve:

$$\pi_t = \kappa \hat{Y}_t. \quad (\text{AS})$$

Here, $\kappa > 0$. To simplify the exposition, we adopt a simpler version of the Phillips curve than the common one in the literature, in which a forward-looking term appears on the right-hand side. This simplifying assumption does not affect anything substantively, as the model focuses on the demand side, with price dynamics playing an auxiliary role in our analysis.

³Table 2 in Appendix A.2.3 focuses on corporate-bond interest rates, while Table 3 in Appendix A.2.3 shows that using government bonds to measure long-term rates does not change the results. Lastly, Table 4 in Appendix A.2.3 demonstrates that the results remain robust using the monetary base, instead of M1, as money measure.

⁴As shown by Abel et al. (1989), it would be possible to introduce sufficient uncertainty in a stochastic endowment economy such that the risk-free rate is negative in an ergodic stochastic steady state. The literature has not explored this possibility in detail and it is much less tractable than assuming $0 < \delta < 1$. For instance, the term δ can arise due to relative wealth in the utility function as in Michaillat and Saez (2021), overlapping generations as in Eggertsson, Mehrotra, and Robbins (2019), deviation from full rationality as in Gabaix (2020), heterogeneity between borrowers and spenders as in Bilbiie (2021), or the heterogeneous-agent New-Keynesian (HANK) literature following Kaplan, Moll, and Violante (2018). Bilbiie (2021) shows a model in which it is possible that $\delta > 1$. Yet, this parametrization would also prevent us from considering a permanent liquidity trap, like in a standard representative-agent model.

This equation is derived under the assumption that firms are monopolistically competitive, with a fixed fraction of firms indexing their prices to the previous period's aggregate price level and the remaining fraction setting prices optimally. Consider the case $\kappa \rightarrow 0$, then all the firms index their prices and $\pi_t = 0$, which is the assumption made by Hicks (1937).

The money-demand (or LM) equation is

$$\hat{M}_t \geq \eta_y \hat{Y}_t - \eta_i \hat{i}_t + \epsilon_t^d \text{ and } \hat{i}_t \geq i_{ZLB}, \quad (\text{LM / ZLB})$$

where $\eta_y > 0$, $\eta_i > 0$, and at least one of the inequalities holds with equality at any given time. ϵ_t^d is a money-demand shock; we set this shock at zero for now and return to it in Section 5. We derive this equation by assuming that real money balances provide transaction services for the household and appear directly in the utility function. At some point, however, households accumulate enough liquidity to satisfy all their transaction needs (that is, they reach satiation, which we denote by the point M^*). The first inequality becomes slack and \hat{i}_t equals i_{ZLB} .

Since we expressed variables in log deviations from steady state, i_{ZLB} is negative, assuming the short-term interest rate cannot go below zero.⁵ The policy maker sets \hat{M}_t via open-market operations in short-term risk-free government bonds and, through this, the short-term nominal interest rate.

The IS equation can be forwarded to yield

$$\hat{Y}_t = -\sigma \mathbb{E}_t \sum_{j=0}^{\infty} \delta^{j+1} (\hat{i}_{t+j} - \pi_{t+j+1} - \hat{r}_{t+j}^e). \quad (4)$$

Equation (4) illustrates that output depends not just on the current real interest rate but also on the expected future path of the real interest rate.

We consider the following assumption about the exogenous disturbance:

Assumption 1. At time 0, $\hat{r}_t^e = \hat{r}_S^e < 0$; it then reverts, with a fixed transition probability μ , in each of the following periods so that $\hat{r}_t^e = 0$ in steady state. We denote the stochastic period in which the shock reverts to a steady state as T . Once the shock reaches a steady state, it stays there forever. We call the periods $t < T$ the short run, denoted S , and the periods $t \geq T$ the long run, denoted L .

One non-conventional property of the Hicks assumption that $\kappa \rightarrow 0$ is that money is not neutral in the long run, which goes against the current consensus in the monetary literature. Long-run monetary non-neutrality, however, is not critical for our result. Instead, what matters is that monetary policy still retains power in the period immediately following the reversal of \hat{r}_t^e to steady state.⁶

⁵Central banks experimented modestly negative interest rates on reserves, with mixed success, because central bank reserves provide some transaction services to the banks which they are willing to pay for (see Rognlie 2016 and Eggertsson et al. 2023).

⁶One can alternatively write the model with a short run ($t < T$), medium run (defined as the stochastic period T), and long run ($t > T$), stipulating that money is neutral in the long run. This extension's results remain unchanged, even if the algebra needs modest adjustment.

We consider two types of monetary-policy regimes:

Assumption 2. Under the *Keynesian policy regime*, $\hat{M}_t = \hat{M}_L = 0$ for $t \geq T$ and $\hat{M}_t = \hat{M}_S$ for $t < T$ is a policy choice. In the case of the Keynesian policy regime, if there is a monetary expansion today, the central bank is expected to reverse it as soon as economic conditions improve (that is, when the shock \hat{r}_S^e reverts to a steady state).

Assumption 3. Under the *monetarist policy regime*, $\hat{M}_t = \hat{M}_S = \hat{M}_L$ for any t is a policy choice. In the case of the monetarist policy regime, people expect any short-run monetary expansion to be permanent.

Under assumptions 1–3, the endogenous variables take on the same value in all periods $t < T$, denoted by the subscript S , and the same value for $t \geq T$, which we denote with the subscript L . The expectation of future output, conditional on $t < T$, is then $\mathbb{E}_t \hat{Y}_{t+1} = \mu \hat{Y}_L + (1 - \mu) \hat{Y}_S$.

The short-run IS equation is then

$$\hat{Y}_S = \frac{\mu\delta}{1 - \delta(1 - \mu)} \hat{Y}_L - \frac{\sigma\delta}{1 - \delta(1 - \mu)} \hat{i}_S + \frac{\sigma\delta}{1 - \delta(1 - \mu)} \hat{r}_S^e, \quad (5)$$

while the LM equation is

$$\hat{M}_S \geq \eta_y \hat{Y}_S - \eta_i \hat{i}_S \text{ and } \hat{i}_S \geq i_{ZLB}. \quad (6)$$

If the LM equation holds with equality in the long run, we can solve for output to yield

$$\hat{Y}_L = \frac{\delta\sigma/\eta_i}{1 - \delta + \sigma\delta(\eta_y/\eta_i)} \hat{M}_L. \quad (7)$$

Hicks (1937) argues that the lower bound on the short-term interest rate, in turn, implies that the long-term interest rate also has a bound greater than zero. Hence, to resolve the controversy, one needs to account for the behavior of the long-term interest rate and the extent to which it is insensitive to a money supply expansion. As we can see in Equation (4), the IS equation depends on the entire expected future path of the short-term nominal interest rate. Also, Equation (5) reveals that output depends on expected future output, or \hat{Y}_L , which in turn is determined by the expectation about the long-term money supply in Equation (7).

What is the effect of a monetary expansion—that is, $\hat{M}_S \uparrow$ —assuming either the Keynesian or the monetarist policy regime? Under both monetary-policy regimes, the LM curve shifts rightward from LM to LM' . Under the Keynesian policy regime, the initial increase in money supply has no further effects. However, the monetarist policy regime unfolds differently. In this regime, the increase in \hat{M}_S also signals a future monetary expansion—specifically, $\hat{M}_L \uparrow$ —once the economy moves beyond the ZLB. This anticipated future expansion does not affect the LM curve. Instead, it shifts the IS curve outward by increasing expectations about future income, stimulating current spending.

Finally, it is straightforward to show that the equilibrium is characterized by

$$\hat{Y}_S = \frac{1}{1 - \delta(1 - \mu)} \left[\frac{(\mu\delta)(\sigma\delta/\eta_i)}{1 - \delta + \sigma(\eta_y/\eta_i)} \hat{M}_L + \sigma\delta(\hat{r}_S^e - i_{ZLB}) \right], \quad (8)$$

with $\hat{M}_L = 0$ under the Keynesian policy regime and $\hat{M}_L = \hat{M}_S$ under the monetarist policy regime. Thus, the monetary expansion is effective under the monetarist regime, while it is not under the Keynesian one.

4 Resolving the Controversy

4.1 Long-Term Interest Rates

The model in Section 3 can be used to price any asset, such as a loan of “infinite duration,” whose price is the relevant interest rate, according to Keynes and Hicks. To capture this idea define the long-term interest rate as the implied yield, denoted by i_t^l , of a perpetuity whose coupon payment declines geometrically at a rate ρ . The implied duration of such a bond is given by $(1 - \rho\beta)^{-1}$, so by appropriate choice of ρ we can approximate a bond of arbitrary duration. The case in which $\rho = 0$ corresponds to a one-period risk-free bond, while $\rho = 1$ is a classic consol.

The long-term interest rate defined by this consol is up to a first-order approximated by

$$\hat{i}_t^l = (1 - \rho\beta/\delta) \mathbb{E}_t \sum_{j=0}^{\infty} \left(\frac{\rho\beta}{\delta} \right)^j \hat{i}_{t+j}, \quad (9)$$

where β is the time-discount factor of the representative household. As suggested by this expression, the long-term interest rate is a weighted average of current and future short-term interest rates. To simplify the analytics, we choose $\rho = \delta^2/\beta$ so that the long-term interest rate corresponds to the yield on a bond with duration $(1 - \delta)^{-1}$.⁷

Under this assumption, we can write the IS equation in the short run as:

$$\hat{Y}_S = -\frac{\sigma\delta}{1 - \delta} (\hat{i}_S^l - \hat{r}_S^{e,l}), \quad (10)$$

where $\hat{r}_t^{e,l} \equiv (1 - \delta) \mathbb{E}_t \sum_{j=0}^{\infty} \delta^j \hat{r}_{t+j}^e$. We can also rewrite the LM equation in terms of the long-term interest rate as we documented that Hicks, and the literature that followed, assumed that money demand depended on the long-term interest rate. As we will show, however, the formulation of the LM equation in terms of the long-term interest rate takes different shapes depending on the assumed monetary-policy regime. We will refer to the resulting relationships as “quasi-money demand,” corresponding to the Keynesian policy regime and the monetarist policy regime.

⁷If we were to further assume that $\delta^2 = \beta$, then this bond would be a classic consol.

To derive a Keynesian quasi-LM curve, we first write the long-term interest rate in the short run as

$$\hat{i}_S^l = \frac{1-\delta}{1-\delta(1-\mu)}\hat{i}_S + \frac{\delta\mu}{1-\delta(1-\mu)}\hat{i}_L. \quad (11)$$

Under the Keynesian policy regime, the interest rate turns to its steady state in the long run so that $\hat{i}_L = 0$. Using Equation (11), the bound on the long-term nominal interest rate is therefore

$$\hat{i}_S^l \geq i_{ZLB,K}^l = \frac{1-\delta}{1-\delta(1-\mu)}i_{ZLB}. \quad (12)$$

Note that $i_{ZLB,K}^l \geq i_{ZLB}$, and the lower bound on the long-term interest rate is higher than the bound on the short-term interest rate if $\mu > 0$. Equation (12) then indicates that people's expectations about future short-term interest rates constrain the long-term interest rate. Under the Keynesian policy regime, people expect short-term interest rates to return to their positive steady-state values once the shock ends. Substituting Equation (11) into the LM equation allows us to derive a Keynesian quasi-LM curve,

$$\hat{M}_S \geq \eta_y \hat{Y}_S - \eta_i \alpha_i^K \hat{i}_S^l \text{ and } \hat{i}_S^l \geq i_{ZLB,K}^l. \quad (13)$$

Here, $\alpha_i^K \equiv \frac{1-\delta(1-\mu)}{1-\delta}$. This equation is a non-structural relationship conditional on the Keynesian policy regime.

Let us now consider a quasi-LM relationship under a monetarist policy regime. The key difference here is that \hat{i}_L no longer remains fixed at its steady-state value because agents expect any increase in the money supply to be permanent—that is, $\hat{M}_L = \hat{M}_S$. Combining Equation (7) with the LM equation, we obtain

$$\hat{i}_L = -\frac{1}{\eta_i + [\sigma\delta/(1-\delta)]\eta_y} \hat{M}_S \text{ if } \hat{M}_S < M^* \text{ and } \hat{i}_L \geq i_{ZLB}. \quad (14)$$

Here, M^* is the money-satiation point in the long run and $\hat{i}_L = i_{ZLB}$ if $\hat{M}_S \geq M^*$.

Under the monetarist policy regime, the ZLB only binds when both i_S and i_L reach their lower bound. Accordingly, from Equation (11), we see that the lower bound on the long-term interest rate under the monetarist regime is the same as that on the short-term interest rate—that is, i_{ZLB} . Using the expression for \hat{i}_L in Equation (14), and using Equation (11) to substitute \hat{i}_S^l for \hat{i}_S in the LM equation, we obtain a monetarist quasi-LM curve:

$$\hat{M}_S \geq \eta_y \alpha_y^M \hat{Y}_S - \eta_i \alpha_i^M \hat{i}_S^l \text{ and } \hat{i}_S^l \geq i_{ZLB}, \quad (15)$$

where $\alpha_y^M \equiv \frac{(1-\delta)\eta_i + \sigma\eta_y}{[1-\delta(1-\mu)]\eta_i + \sigma\eta_y}$ and $\alpha_i^M \equiv \frac{1-\delta(1-\mu)}{1-\delta} \alpha_y^M$.⁸

⁸We derived all the relationships above assuming that the interest rate measure is a risk-free rate. Analogous relationships can be derived using an interest rate which incorporates a risk of default, see Appendix A.3.

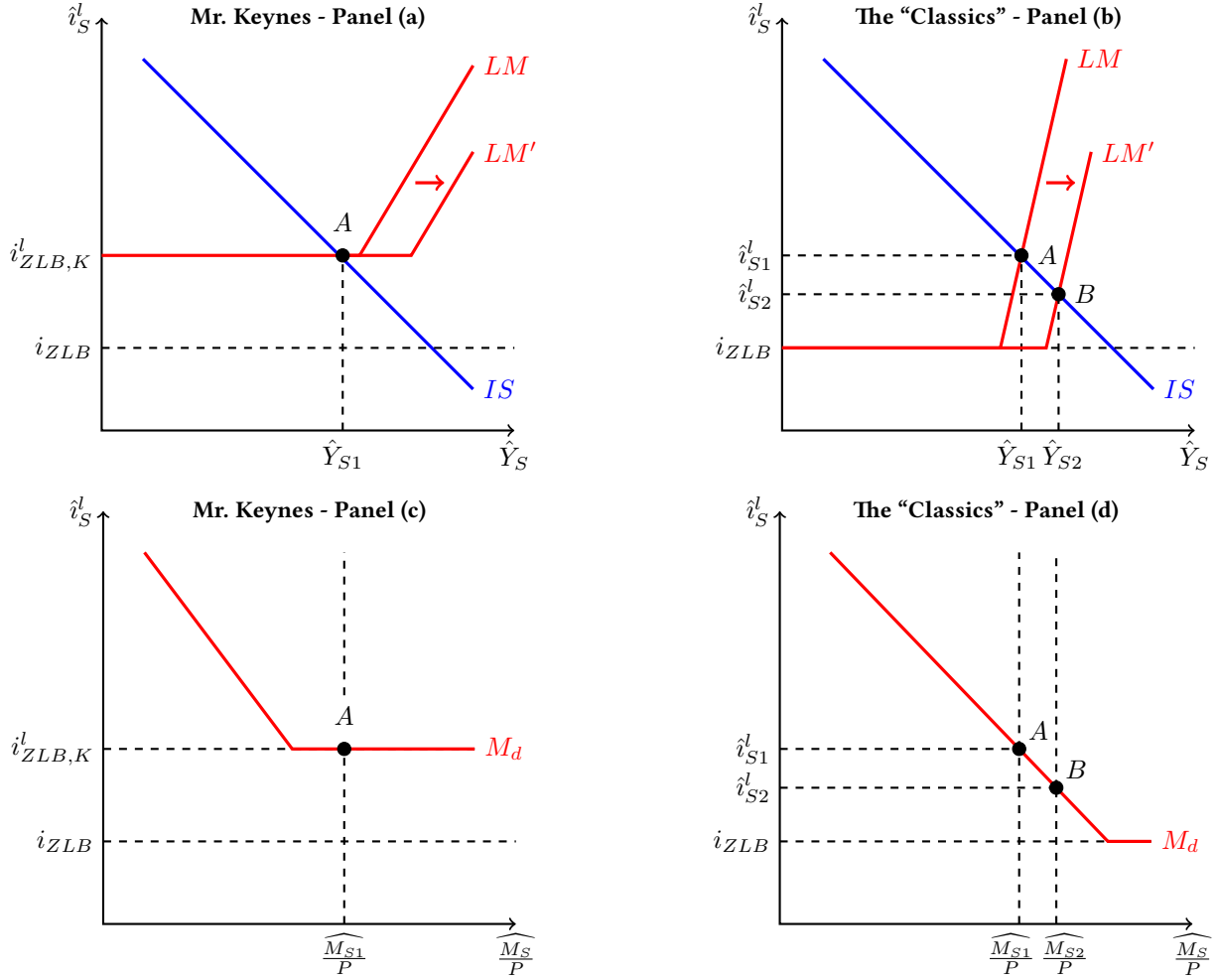


Figure 3: Suggested Reconciliation

4.2 Discussion

The IS and LM equations under the two monetary-policy regimes are shown in Figure 3 and replicate the original figure from Hicks (1937). However, the money-demand equations written in terms of the long-term interest rate are not structural, in the sense of Lucas (1976), but instead depend upon the assumed regime. Under what condition is the flat region of the LM curve reached under the regimes?

In the Keynesian regime, a sufficient condition is that the current short-term interest rate, \hat{i}_S , reaches the lower bound. The future short-term interest rate expectation once the shock is over, \hat{i}_L , is fixed at the steady-state value, implying the term structure will always be upward-sloping. Significantly, however, the lower bound on the long-term interest rate, $i_{ZLB,K}^l$, depends on the expected shock duration that gives rise to the ZLB. Hence, if people expect the shock to last longer, this will automatically reduce $i_{ZLB,K}^l$ and make the term structure interest rates flatter.

In the monetarist regime, however, both \hat{i}_S and \hat{i}_L need to reach the ZLB for the policymakers to find themselves in the flat region of the LM curve, implying that the short-term interest rate needs to reach zero and that the entire term structure is flat, i.e., both long and short rates. Thus, as often argued by monetarists, a “true” liquidity trap corresponds to the case in which the whole yield curve is flat—that is, both long- and short-term rates are zero.

We can thus reformulate the controversy based on our analysis as follows. The Keynesians asserted that central banks could not lower long-term interest rates further once the short-term interest rate reached zero. This assertion implicitly assumes that central banks cannot commit to future monetary expansion: an increase in today’s money supply does not signal future monetary expansion. In contrast, the monetarists asserted that central banks could always lower long-term interest rates, even when the short-term interest rate reached zero, provided long-term interest rates remained positive. They argued that central banks needed to increase the money supply to achieve this. In our model, this assertion implicitly assumes that when central banks increase the money supply today, they credibly signal future loosening monetary policy.

5 Identifying Quasi-Money Demand

This section demonstrates how two assumptions allow us to identify the quasi-money demand. We illustrate how to achieve identification by showing an alternative regression to those proposed in the previous literature. While we use a less flexible functional form for this regression compared to Section 2, this approach clarifies how we can identify quasi-money demand, as we derive it directly from the theoretical model in Section 3.

The literature surveyed in Section 2 assumed a well-defined money demand which is “stable.” As stressed by Friedman (1956), “Quantity theorists accept the empirical hypothesis that the demand for money is highly stable.” Here, the term “stable” refers to the assumption that shocks to money demand are not “too important” as long as the period under consideration is sufficiently long. Indeed, seasonal fluctuations in money demand were already well recognized, so the literature focused on annual data. The monetarist claim that money demand was “stable” then suggested that variations in money demand do not play an important role, when considering data at an annual frequency. Thus, the literature we replicated in Section 2 implicitly assumed that it was instead the supply of money that was moving around over time. With monetary stock being the primary impulse, this implied that that data should be tracing out the demand for money absent other shocks.

Using the theoretical model of Section 3, we can apply this argument to identify the quasi-money demand by making two assumptions. First, suppose the natural rate of interest \hat{r}_t^e is the primary shock perturbing the economy. Second, imagine that the central bank determines the money supply under optimal policy under discretion. This second assumption implies that the money supply is set to stabilize the output gap whenever possible, that is implementing the Keynesian policy regime.

Relative to our earlier assumption, we incorporate two additional sources of shocks to show how to identify quasi-money demand: a money-demand shock, $\epsilon_{S,t}^d$, and a time-varying transition probability, μ_t , conditional on the ZLB being binding. The money-demand shock— $\epsilon_{S,t}^d$ —follows the same two-state Markov process as the natural rate of interest— $\hat{r}_{S,t}^e$ —reverting with probability μ_t to the absorbing steady state. We assume that the exogenous variables are not correlated with one another in the short run but satisfy the martingale property—that is, $\mathbb{E}_t \hat{r}_{S,t+1}^e = \hat{r}_{S,t}^e$, $\mathbb{E}_t \epsilon_{S,t+1}^d = \epsilon_{S,t}^d$, and $\mathbb{E}_t \mu_{t+1} = \mu_t$. Following the same steps, we obtain the expression for the Keynesian quasi-LM curve. The crucial difference is that the variables vary stochastically. If $\hat{i}_{S,t}^l \geq i_{ZLB,K}^l$, then

$$\hat{M}_{S,t} = \eta_y \hat{Y}_{S,t} - \eta_i \alpha_i^K \hat{i}_{S,t}^l + \epsilon_{S,t}^d, \quad (16)$$

where $\alpha_i^K \equiv \frac{1-\delta(1-\mu)}{1-\delta}$.⁹ Since $\hat{Y}_{S,t} = 0$, we can rearrange the equation to obtain

$$\hat{i}_{S,t}^l = -(\eta_i \alpha_i^K)^{-1} \hat{M}_{S,t} + (\eta_i \alpha_i^K)^{-1} \epsilon_{S,t}^d. \quad (17)$$

The model satisfies this equation only if $\hat{i}_{S,t}^l$ equals or exceeds $i_{ZLB,K}^l$. However, $i_{ZLB,K}^l$ no longer remains fixed, that is:

$$i_{ZLB,K}^l = \frac{1-\delta}{1-\delta(1-\mu_t)} i_{ZLB}^l, \quad (18)$$

This value may change since we allow μ_t to time vary when the ZLB is binding. Equation (18) implies that the ZLB on the long-term nominal interest rate depends on the expected duration of the ZLB on the short-term rate, μ_t .

Since $\hat{i}_{S,t}^l \equiv \log(1 + i_{S,t}^l) - \log(1 + \bar{i}^l)$, Equations (17) and (18) can be summarized by a simple regression:

$$\log(1 + i_t^l) = \beta_0 + \beta_1 D_t \log(M_t/Y_t) + \beta_2 D_t + \xi_t \quad (19)$$

Here, i_t^l is the long-term nominal interest rate in levels, M_t is money in dollar value, Y_t is nominal income, and ξ_t is an exogenous disturbance. The role of nominal income is to express the money supply in real terms. We use a dummy variable, D_t , that takes the value 0 when the ZLB is binding, in which case Equation (18) applies, and the value 1 when the ZLB is not binding, in which case Equation (17) applies.

Table 6 in Appendix A.5 reports the results using 1900–2019 data from the bottom panel of Figure 2.¹⁰ The main finding is an estimate of β_0 which is statistically different from zero with a 99 percent confidence interval from 1.19 to 1.48. This parameter is the constant of the regression when the ZLB is binding (i.e., $D_t = 0$). It represents the estimated $i_{ZLB,K}^l$ in log units. Converted to a long-term nominal interest rate, it suggests a ZLB on the long-term interest rate of 2.82 percent with a 99 percent confidence interval from 2.29 to 3.39 percent.

⁹Conditional on $t < T$, we once again have that $\mathbb{E}_t \hat{Y}_{t+1} = \mu_t \hat{Y}_L + (1 - \mu_t) \hat{Y}_{S,t} = (1 - \mu_t) \hat{Y}_{S,t}$ by using the martingale property.

¹⁰We posit that the ZLB is binding when the effective federal funds rate is below 0.25, corresponding to 1934–36, 1938–41, and 2009–15.

We present the piecewise linear regression line (in logs) using the estimated values from Table 6 in Appendix A.5 in the top panel of Figure 4. This presentation helps us understand why the two assumptions correctly identify quasi-money demand. We assume that the natural rate of interest— \hat{r}_t^e —is the main shock perturbing the economy and that the central bank sets the money supply to offset these shocks (as the Keynesian policy regime implies). If only shocks to the natural interest rate occur, and if the central bank offsets them by changing the money supply, then the regression will trace out the quasi-money demand that the model suggests, as the black dots show.

Let us now consider money-demand shocks— $\epsilon_{S,t}^d$ —and how they affect the estimated quasi-money demand. On the upward-sloping part of the quasi-money demand curve, a shock to money demand moves the data away from the estimated red line. We show one standard deviation of the residuals from the estimated regression as a band around the black dot on the far left of the figure. According to our model, the residuals of the regression when $D_t = 1$ represent shocks to money demand. A low standard deviation suggests evidence supporting the hypothesis that money demand shocks are sufficiently small to identify quasi-money demand correctly.

Now, let us consider the flat part of the quasi-money demand curve. In this segment, we cannot determine the demand for money because money and government bonds with zero interest rates are perfect substitutes, implying that a shock to the demand for money produces no effect. However, as Equation (18) shows, changes in people’s beliefs about how long the ZLB will remain binding, governed by μ_t for the short-term nominal interest rate, shift the estimated $i_{ZLB,K}^l$ —that is, the lower bound of the long-term nominal interest rate. We interpret the residuals of the estimated regression when $D_t = 0$ as representing changes in the expected duration of the ZLB. We display one standard deviation of the regression residual when $D_t = 0$ is the band on the far right of the figure.

In the second part of Figure 4, we present the piecewise linear regression line (instead of its logarithmic counterpart) that our estimation implies, comparing it against the actual data. The logarithmic functional form of the regression, which is more restrictive than the estimation procedure in Section 2, challenges the fit. Indeed, this restriction doesn’t fully account for the curve’s steepness at higher interest rates. A higher-order approximation of the model might capture this nonlinearity. We leave this for future work, as it is not essential to interpreting the results.

Our model analysis fundamentally changes how we interpret the finding that the estimated $i_{ZLB,K}^l$ exceeds zero. This positive number does not prove that monetary policy cannot stimulate aggregate demand. Instead, it is evidence of a Keynesian policy regime, which suggests that even if central banks increase the money supply at the ZLB, markets expect them to contract it as soon as the natural rate of interest recovers. Accordingly, we believe the most plausible interpretation of our finding suggests that expectations aligned approximately with a Keynesian regime during the sample period. Large increases in money supply at the ZLB do not significantly raise expectations of higher future money supply and, consequently, do not provide any additional increase in demand on their own.

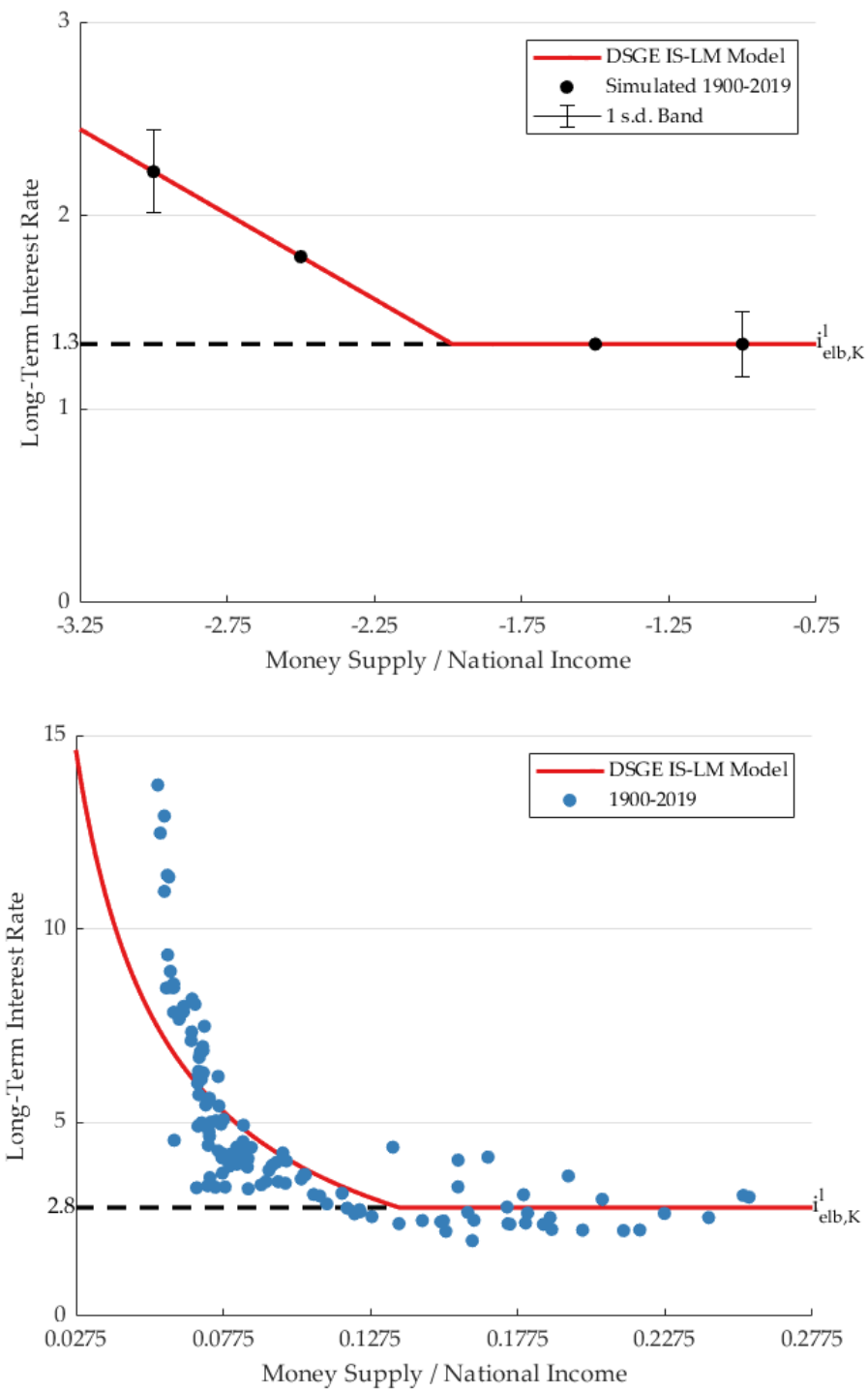


Figure 4: Long-Term Government-Bond Interest Rates versus Money Supply over National Income

6 Conclusion

Viewed through the prism of structural quasi-money demand, the economy has behaved as if following a Keynesian regime in 1900-2019. The massive increase in the monetary base—tripling in 2008-2015—did little to move output and prices. While Keynesians may have won a narrow empirical battle, based on annual data and quasi-money demand, they ultimately lost the broader theoretical war. The monetarist assertion that monetary policy remains effective at the ZLB aligns with modern consensus, contrary to the Keynesian view of monetary policy as “pushing on a string.” As surveyed by Eggertsson and Egiev (2024), the literature finds that the 1933 monetary-policy regime change, supported by fiscal measures, was instrumental in ending the Great Depression. Similarly, substantial evidence shows that forward guidance and quantitative easing (QE) had significant impacts following the Great Recession (see Woodford, 2012 and Eggertsson and Egiev, 2024).

However, the legacy of monetarism remains mixed. The modern understanding of transmission differs from the monetarist “quantity equation” framework. Rather than operating through static movements in money supply, affecting long-term rates via non-structural money demand, modern theory emphasizes the management of expectations about future policy. At the ZLB, these expectations become paramount, with expectations of future short-term rate serving as the primary indicator of policy stance. Financial frictions may also play a role, for instance via QE, but not through a mechanical relationship between money demand and supply.

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Online Appendix for
Mr. Keynes and the “Classics”; A Suggested Reconciliation

Appendix A.2 - Empirical Estimates for the Liquidity Trap

A.2.2 - Data and Estimation Methods

A.2.2.1 - Data

Table 1: Data Sources

(1900-1958)		(1959-2019)
M1 (M_t)	<i>Historical Statistics of the United States</i> ¹	Federal Reserve Economic Data ¹
Corporate Interest Rates (i_t)	<i>Historical Statistics of the United States</i> ²	<i>Historical Statistics of the United States</i> and Federal Reserve Economic Data ²
National Income (Y_t)	<i>Historical Statistics of the United States and Federal Reserve Economic Data</i> ³	Federal Reserve Economic Data ³
Assets (A_t)	<i>A Study of Saving in the United States</i> and Board of Governors of Federal Reserve System ⁴	Board of Governors of Federal Reserve System ⁴
Government Interest Rates (i_t)	Jordà-Schularick-Taylor Macrohistory Database and Federal Reserve Economic Data ⁵	Federal Reserve Economic Data ⁵
Monetary Base (M_t)	Jordà-Schularick-Taylor Macrohistory Database ⁶	Jordà-Schularick-Taylor Macrohistory Database and Federal Reserve Economic Data ⁶

¹From 1899 to 1957, Series X 267 in U.S. Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1957*, Washington, D.C., 1960—original source of Pifer (1969) and White (1972); for 1958, Series Cj49 in *Historical Statistics of the United States, Millennial Edition Online*, Cambridge University Press, Cambridge (England), 2006; from 1959 to 2019, 'M1SL' Series in Federal Reserve Economic Data.

²From 1900 to 1957, Series X 346 (20-year maturity) in U.S. Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1957*, Washington, D.C., 1960—original source of Pifer (1969) and White (1972); from 1958 to 1975, Series Cj1241 (20-year maturity) in *Historical Statistics of the United States, Millennial Edition Online*, Cambridge University Press, Cambridge (England), 2006; from 1975 to 2019, 'AAA' Series (20-year-and-above maturity) in Federal Reserve Economic Data.

³From 1900 to 1957, Series F 7 in U.S. Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1957*, Washington, D.C., 1960; from 1958 to 2019, 'NICUR' Series in Federal Reserve Economic Data.

⁴From 1900 to 1949, the series has been constructed following Meltzer (1963a), which is the original source of Pifer (1969) and White (1972). "Total assets (A) were taken from Goldsmith, [R. W. Goldsmith, *A study of Savings in the United States*, Princeton University Press, Princeton NJ., 1956], Tables W-9 through W-16, and were available for eight benchmark dates from 1900-1949. The series used in the estimates for 1900-1945 was obtained by taking the ratio of total assets, A, to wealth, G (from Table W-1). Denote this ratio for any base year as b_0 . Let b_n = the next base year ratio, and let $i = 0, 1, \dots, n$ be an index of the years between base year dates inclusive of the end points. Then $A_i = \left[\frac{(n-i)}{n} b_0 + \frac{i}{n} b_n \right]$ "; from 1950 to 2019, the series has been built summing the Series FL10200005.A, FL112000005.A, FL152000005.A, FL212000095.A, FL312000095.A, and FL792000095.A in the Financial Accounts of Board of Governors of Federal Reserve System.

⁵From 1900 to 1953, 'lrate' Series (10-year maturity) in Jordà-Schularick-Taylor Macrohistory Database; from 1954 to 2019, 'GS20' Series (20-year maturity) in Federal Reserve Economic Data.

⁶From 1900 to 2016, 'narrowm' Series in Jordà-Schularick-Taylor Macrohistory Database; from 2017 to 2019, 'BOGMBASE' Series in Federal Reserve Economic Data.

A.2.2.1 - Pifer (1969)'s Two-Step Maximum Likelihood Estimation

Pifer (1969) proposes the following nonlinear two-step maximum likelihood method to estimate i_{min} :

$$\begin{aligned}
 \max_{i_{min}, \gamma, \omega, \alpha} \quad & \mathcal{L}_p(y; i_{min}, \gamma, \omega, \alpha) = \max_{i_{min}} \left[\max_{\gamma, \omega, \alpha} \mathcal{L}_p(y; \gamma, \omega, \alpha | i_{min}) \right] \\
 s.t. \quad & \mathcal{L}_p(y; \gamma, \omega, \alpha | i_{min}) = -e' e, \\
 & y = \log(M_t) = \log(\gamma) + \omega \log(Y_t) + \alpha \log(i_t - i_{min}) + \xi_t.
 \end{aligned}$$

The first step is constructing a grid for i_{min} and, for each possible value of i_{min} , running the above regression to calculate the sum of squared residuals.¹¹ The second step is minimizing such sum, that is maximizing the likelihood function \mathcal{L}_p , to identify the maximum likelihood estimate of i_{min} .

¹¹We construct a grid for i_{min} from -1.5 to the minimum value of the series of long-term government-bond (corporate-bond) interest rates, in increments of 0.01.

A.2.2.2 - White (1972)'s Two-Step Maximum Likelihood Estimation

White (1972) proposes the following generalization of Pifer's estimation method to estimate i_{min} :

$$\begin{aligned} \max_{\lambda, i_{min}, \gamma, \omega, \alpha} \quad & \mathcal{L}_w(y; \lambda, i_{min}, \gamma, \omega, \alpha) = \max_{\lambda, i_{min}} \left[\max_{\gamma, \omega, \alpha} \mathcal{L}_w(y; \gamma, \omega, \alpha | \lambda, i_{min}) \right] \\ \text{s.t.} \quad & \mathcal{L}_w(y; \gamma, \omega, \alpha | \lambda, i_{min}) = -\frac{T}{2} \log(e'e) + (\lambda - 1) \sum_{t=1}^T \log(M_t), \\ & y = \frac{M_t^\lambda - 1}{\lambda} = \frac{\gamma^\lambda - 1}{\lambda} + \omega \frac{Y_t^\lambda - 1}{\lambda} + \alpha \frac{(i_t - i_{min})^\lambda - 1}{\lambda} + \xi_t. \end{aligned}$$

The first step is constructing a two-dimensional grid for λ and i_{min} and, for each possible point in this grid, running the above regression to calculate the corresponding likelihood function \mathcal{L}_w .¹² The second step is maximizing the \mathcal{L}_w function to identify the maximum likelihood estimate of i_{min} .

A.2.3 - Empirical Results

Table 2: Estimates of i_{min} and λ using M1 and Long-Term Corporate Interest Rates

Time Period		(1900-1958)		(1900-2019)		
Specification	Pifer (1969)	White (1972)		Pifer (1969)	White (1972)	
	i_{min}	λ	i_{min}	i_{min}	λ	i_{min}
i_t, Y_t	2.17	0.18	2.33	2.30	-0.10	2.25
	(0.01, 2.33) ¹	(-0.09, 0.35) ³	(0.67, 2.34) ³	(2.17, 2.33) ¹	(-0.16, -0.05) ³	(2.02, 2.32) ³
	(-1.50, 2.34) ²	(-0.16, 0.39) ⁴	(-1.50, 2.34) ⁴	(2.09, 2.34) ²	(-0.17, -0.03) ⁴	(1.93, 2.33) ⁴
i_t, Y_t, A_t	2.24	-0.36	1.70	2.30	-0.24	2.23
	(1.94, 2.32) ¹	(-0.60, -0.12) ³	(-0.94, 2.21) ³	(2.17, 2.34) ¹	(-0.28, -0.19) ³	(2.10, 2.29) ³
	(1.69, 2.33) ²	(-0.66, -0.05) ⁴	(-1.50, 2.27) ⁴	(2.09, 2.34) ²	(-0.30, -0.18) ⁴	(2.05, 2.30) ⁴

¹ 95% and ² 99% confidence intervals; see Hayashi (2000, pp. 52-53); ³ 95% and ⁴ 99% confidence intervals; see Greene (2018, p. 554).

¹² Respectively, we construct a two-dimensional grid for λ and i_{min} from -1.5 to 1.5 (excluding 0) and from -1.5 to the minimum value of the series of long-term government-bond (corporate-bond) interest rates, in increments of 0.01.

Table 3: Estimates of λ and i_{min} using M1 and Long-Term Government Interest Rates

Specification	$\frac{M_t^\lambda - 1}{\lambda} = \frac{\gamma^\lambda - 1}{\lambda} + \omega \frac{Y_t^\lambda - 1}{\lambda} + \alpha \frac{(i_t - i_{min})^\lambda - 1}{\lambda} + \delta \frac{A_t^\lambda - 1}{\lambda} + u_t$	
	λ	i_{min}
Government Interest Rates	-0.19 (-0.27, -0.08) ¹ (-0.29, -0.04) ²	1.72 (1.36, 1.87) ¹ (1.20, 1.89) ²

¹95% and ²99% confidence intervals; see Greene (2018, p. 554). The estimated coefficients with government interest rates are: *constant* = -1.69 (0.261), $\hat{\omega}$ = 0.80 (0.093), $\hat{\alpha}$ = -0.10 (0.008), $\hat{\delta}$ = 0.49 (0.141); standard errors are in square brackets. The estimated coefficients with corporate interest rates are: *constant* = -3.09 (0.202), $\hat{\omega}$ = 0.50 (0.066), $\hat{\alpha}$ = -0.08 (0.004), $\hat{\delta}$ = 1.21 (0.113); standard errors are in square brackets.

Table 4: Estimates of λ and i_{min} using the Monetary Base

Specification	$\frac{M_t^\lambda - 1}{\lambda} = \frac{\gamma^\lambda - 1}{\lambda} + \omega \frac{Y_t^\lambda - 1}{\lambda} + \alpha \frac{(i_t - i_{min})^\lambda - 1}{\lambda} + \delta \frac{A_t^\lambda - 1}{\lambda} + u_t$	
	λ	i_{min}
Government Interest Rates	-0.07 (-0.12, -0.03) ¹ (-0.13, -0.02) ²	1.69 (1.35, 1.84) ¹ (1.21, 1.86) ²
Corporate Interest Rates	-0.13 (-0.18, -0.08) ¹ (-0.19, -0.07) ²	2.13 (1.84, 2.26) ¹ (1.72, 2.28) ²

¹95%, and ²99% confidence intervals, see Greene (2018, p. 554). The estimated coefficients with government interest rates are: *constant* = -4.16 (0.281), $\hat{\omega}$ = 0.48 (0.124), $\hat{\alpha}$ = -0.35 (0.017), $\hat{\delta}$ = 0.83 (0.138); standard errors are in square brackets. The estimated coefficients with corporate interest rates are: *constant* = -6.47 (0.332), $\hat{\omega}$ = -0.02 (0.130), $\hat{\alpha}$ = -0.26 (0.013), $\hat{\delta}$ = 1.89 (0.169); standard errors are in square brackets.

Appendix A.3 - A Classic Lucas Critique in a New-Keynesian IS-LM Model

A.3.1 - Households

There is a continuum of identical households of measure 1. Household j solves the following problem:

$$\begin{aligned}
 \max_{\{C_t(j), M_t(j), B_t(j), B_t^r(j), B_t^l(j), N_t(j)\}_{t=0}^{\infty}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t(j)) + \chi \left(\frac{M_t(j)}{P_t} \right) \xi_t^d + w \left(\frac{A_t(j) - A_t}{P_t} \right) - v(N_t(j)) \right] \xi_t \\
 \text{s.t.} \quad & P_t C_t(j) + M_t(j) + B_t(j) + B_t^r(j) + S_t B_t^l(j) = \\
 & = W_t N_t(j) + M_{t-1}(j) + (1 + i_{t-1}) B_{t-1}(j) + (1 - \omega_t)(1 + i_{t-1}^r) B_{t-1}^r(j) + \\
 & + (1 + \rho S_t) B_{t-1}^l(j) + \int_0^1 Z_t(i, j) di, \\
 & A_t(j) = M_t(j) + B_t(j) + B_t^r(j) + S_t B_t^l(j), \\
 & i_t \geq 0.
 \end{aligned}$$

Here, β is an intertemporal discount factor, $C_t(j) \equiv \left[\int_0^1 c_t(i, j)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$ is the aggregate consumption, $M_t(j)$ is the amount of dollars that the household holds at time t , $B_t(j)$ is a risk-free nominal bond that pays i_t numbers of dollars at time $t+1$, $B_t^r(j)$ is a nominal bond that pays i_t^r number of dollars at time $t+1$, but with probability ω_{t+1} it will not be repaid, $B_t^l(j)$ is a perpetuity that pays p^j dollars in period $j+1$ and S_t is its price, $N_t(j)$ is the labor supply that the household offers, ξ_t^d is a money demand shock, and ξ_t is a preference shock.

The function u is the period utility of consumption, it is increasing and concave in its argument and at least twice differentiable. The function χ denotes the period utility of holding real money balances, it is increasing and concave in its argument. Define the real money balance $m_t \equiv \frac{M_t}{P_t}$, we assume that there is satiation at m^* so that, the partial derivative of the function χ , $\chi_m(m_t) = 0$ for $m_t \geq m^*$. The function w represents the period utility that the household has from its asset holding, $A_t(j)$, relative to the aggregate asset holding in the economy A_t , as in Michaillat and Saez (2021); it is increasing and concave in its argument and at least twice differentiable.

Finally, $P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$ denotes the aggregate price index, W_t denotes the nominal wage rate, and $Z_t(i)$ are the profits of firm i . All households hold the same assets in equilibrium so that $A_t(j) = A_t = 0$. We substitute this equilibrium condition in the optimality conditions below, to omit reference to j .

We can solve the household problem by formulating a Lagrangian. Combining the first-order conditions of the Lagrangian results in the Euler, the money demand, the asset pricing, and the labor supply equations:

$$u_c(C_t) = \beta(1 + i_t) \mathbb{E}_t \left[u_c(C_{t+1}) \frac{\xi_{t+1}}{P_{t+1}} \right] \frac{P_t}{\xi_t} + w_A(0), \quad (\text{EE})$$

$$\frac{\chi_m(m_t)}{u_c(C_t)} \xi_t^d \geq \frac{i_t}{(1+i_t)} - \frac{i_t}{(1+i_t)} \frac{w_A(0)}{u_c(C_t)}; i_t \geq 0, \quad (\text{MD})$$

$$u_c(C_t) = \beta(1+i_t^r) \mathbb{E}_t \left[(1-\omega_{t+1}) u_c(C_{t+1}) \frac{\xi_{t+1}}{P_{t+1}} \right] \frac{P_t}{\xi_t} + w_A(0), \quad (\text{AP1})$$

$$u_c(C_t) = \beta \mathbb{E}_t \left[(1+\rho S_{t+1}) u_c(C_{t+1}) \frac{\xi_{t+1}}{P_{t+1}} \right] \frac{P_t}{\xi_t} + w_A(0), \quad (\text{AP2})$$

$$\frac{v_N(N_t)}{u_C(C_t)} = \frac{W_t}{P_t}. \quad (\text{LS})$$

A.3.2 - Asset Pricing of the Perpetuity

We define the duration of the perpetuity

$$D \equiv \frac{\beta \sum_{j=0}^{\infty} (j+1)(\beta\rho)^j}{\beta \sum_{j=0}^{\infty} (\beta\rho)^j}, \quad (20)$$

while its yield at time t is the interest rate i_t^l which solves the equation:

$$S_t = \sum_{j=0}^{\infty} \frac{\rho^j}{(1+i_t^l)^{j+1}}, \quad (21)$$

implying that:

$$1 + i_t^l = S^{-1} + \rho. \quad (\text{AP3})$$

A.3.3 - Firms

There is one firm for each good i which faces the demand function $y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t$, where Y_t denotes aggregate output. The optimal spending decision of the households across good types implies the demand function for each firm. We assume that a fixed fraction of firm γ sets their prices flexibly, while the remaining fraction $1 - \gamma$ indexes their prices to the past price level. The flexible-price firm i maximize profits at time t :

$$\begin{aligned} \max_{p_t(i), N_t(i)} \quad & Z_t(i) = p_t(i) y_t(i) - W_t N_t(i) \\ \text{s.t.} \quad & y_t(i) = N_t(i), \\ & y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t. \end{aligned}$$

We obtain the optimal pricing for firm i from the first-order condition of the profit maximization and, assuming a symmetric equilibrium, so that $p_t(i) = p_t^{flex}$:

$$\frac{p_t^{flex}}{P_t} = \frac{\theta}{\theta - 1} \frac{W_t}{P_t}.$$

The firms that index their prices set

$$p_t^{index} = P_{t-1}.$$

The price index at time t can now be written as:

$$P_t = \left[\gamma (p_t^{flex})^{1-\theta} + (1 - \gamma) (p_t^{index})^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

Since production is linear in labor, aggregate hours are given by

$$N_t = \int_0^1 y_t(i) di = Y_t \int_0^1 \left(\frac{p_t(i)}{P_t} \right)^{-\theta} di = Y_t \Delta_t,$$

where $\Delta_t \equiv \int_0^1 \left(\frac{p_t(i)}{P_t} \right)^{-\theta} di$.

The labor supply can now be expressed as

$$\frac{v_N(Y_t \Delta_t)}{u_C(C_t)} = \frac{W_t}{P_t}.$$

Using this equation and the expressions for p_t^{flex} and p_t^{index} , we obtain the price dispersion:

$$\Delta_t = \gamma \left(\frac{\theta}{\theta - 1} \frac{v_N(Y_t \Delta_t)}{u_C(C_t)} \right)^{-\theta} + (1 - \gamma) (\Pi_t^{-1})^{-\theta}, \quad (\text{PD})$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$.

Similarly, following the same steps, we can use the price index to state a non-linear Phillips curve:

$$1 = \gamma \left(\frac{\theta}{\theta - 1} \frac{v_N(Y_t \Delta_t)}{u_C(C_t)} \right)^{1-\theta} + (1 - \gamma) (\Pi_t^{-1})^{1-\theta}. \quad (\text{PC})$$

A.3.4 - Market Clearing

Assume that all production is consumed:

$$Y_t = C_t. \quad (\text{MC})$$

A.3.5 - Equilibrium Definition in the Non-Linear Model

An equilibrium is a collection of stochastic processes for $\{C_t, P_t, i_t, i_t^r, S_t, i_t^l, \Delta_t, Y_t\}$ that solve the Euler equation (EE), the money demand equation (MD), the asset pricing equations (AP1, AP2, AP3), the price dispersion equation (PD), the Phillips curve (PC), and the market clearing equation (MC) given the exogenous shocks $\{\xi_t^d, \xi_t, \omega_t\}$ and a policy specification for the sequence $\{M_t\}$.

A.3.6 - Steady State

We consider a steady state in which inflation is zero, that is, $\bar{\Pi}_t = 1$, and there is no price dispersion, that is, $\Delta_t = 1$. Steady-state output, denoted by \bar{Y} , then solves:

$$\frac{v_N(\bar{Y})}{u_C(\bar{Y})} = \frac{\theta - 1}{\theta}.$$

Define $\delta \equiv 1 - \frac{w_A(0)}{u_C(\bar{Y})}$, we assume that $u_C(\bar{Y}) > w_A(0) \geq 0$, so that $0 < \delta \leq 1$.

The Euler equation (EE) in steady state implies that

$$1 + \bar{i} = \beta^{-1}\delta.$$

We assume that the steady-state interest rate is positive, which implies the restriction that $\delta > \beta$.

Let us denote by $\bar{\omega}$ the “steady-state” value of ω_t ; the steady-state risky interest rate is implied by the Euler equation (EE) and the first asset pricing equation (AP1):

$$1 + \bar{i}^r = \frac{1 + \bar{i}}{1 - \bar{\omega}} = \frac{\beta^{-1}\delta}{1 - \bar{\omega}}.$$

The Euler equation (EE) and the second asset pricing equation (AP2) imply that:

$$\bar{S} = \frac{1}{1 + \bar{i} - \rho} = \frac{1}{\beta^{-1}\delta - \rho}.$$

The money demand equation (MD) can be used to solve for the steady-state real money balance \bar{m} :

$$\frac{\chi_m(\bar{m})}{u_c(\bar{Y})} \bar{\xi}^d \geq \frac{\bar{i}}{(1 + \bar{i})} - \frac{\bar{i}}{(1 + \bar{i})} \frac{w_A(0)}{u_c(\bar{Y})}; \bar{i} \geq 0.$$

Finally, from the second and third asset pricing equations (AP2, AP3), we obtain:

$$(1 + \bar{i}^l) = \frac{1 + \rho\bar{S}}{\bar{S}} = (1 + \bar{i}).$$

A.3.7 - Log-Linear Approximation

We define the elasticity of intertemporal substitution $\sigma \equiv -\frac{u_C(\bar{Y})}{u_{CC}(\bar{Y})\bar{Y}} > 0$ and an approximation of the Euler equation (EE) yields the IS curve:

$$\hat{Y}_t = \delta \mathbb{E}_t \hat{Y}_{t+1} - \sigma \delta (\hat{i}_t - \mathbb{E}_t \pi_{t+1} - \hat{r}_t^e), \quad (\text{IS})$$

where $\hat{r}_t^n \equiv \hat{\xi}_t - \mathbb{E}_t \hat{\xi}_{t+1}$, with $\hat{\xi}_t \equiv \log \xi_t - \log \bar{\xi}$, and with the other variables defined as $\hat{Y}_t \equiv \log Y_t - \log \bar{Y}$, $\pi_t \equiv \log P_t - \log P_{t-1}$, and $\hat{i}_t \equiv \log(1 + i_t) - \log(1 + \bar{i})$.

Approximating the Euler equation (EE) and the first asset pricing equation (AP1) yields:

$$\hat{i}_t = \hat{i}_t^r - \mathbb{E}_t \hat{\omega}_{t+1}, \quad (\text{AP1}^*)$$

where $\hat{i}_t^r = \log(1 + i_t^r) - \log(1 + \bar{i}^r)$ and $\hat{\omega}_{t+1} = \frac{\omega_t - \bar{\omega}}{1 - \bar{\omega}}$.

Approximating the Euler equation (EE) and the second asset pricing equation (AP2) yields:

$$\hat{S}_t = \frac{\beta \rho}{\delta} \mathbb{E}_t \hat{S}_{t+1} - \hat{i}_t, \quad (\text{AP2}^*)$$

where $\hat{S}_t = \log S_t - \log \bar{S}$.

Approximating the second asset pricing equation (AP2) yields:

$$\hat{S}_t = -\frac{\delta}{\delta - \beta \rho} \hat{i}_t^l, \quad (\text{AP3}^*)$$

where $\hat{i}_t^l = \log(1 + i_t^l) - \log(1 + \bar{i}^l)$.

Using the last two equations (AP2*, AP3*), we then obtain a relation between \hat{i}_t^l and all the series of short-term risk-free interest rates under the assumption that $\rho < \frac{\delta}{\beta}$:

$$\hat{i}_t^l = (1 - \rho\beta/\delta) \mathbb{E}_t \sum_{j=0}^{\infty} \left(\frac{\rho\beta}{\delta} \right)^j \hat{i}_{t+j}.$$

We define the elasticity of real money balances as $\psi \equiv -\frac{\chi_m(\bar{m})}{\chi_{mm}(\bar{m})\bar{m}} > 0$ and an approximation of the money demand equation (MD) yields the LM curve:

$$\hat{M}_t \geq \eta_y \hat{Y}_t - \eta_i \hat{i}_t + \epsilon_t^d \text{ and } \hat{i}_t \geq i_{ZLB}, \quad (\text{LM} / \text{ZLB})$$

where $\eta_y \equiv \psi \sigma^{-1} \frac{\bar{i}}{\bar{i} + 1 - \delta} \geq 0$, $\eta_i \equiv \psi \frac{\beta}{1 - \beta} \geq 0$, $\hat{m}_t \equiv \log m_t - \log \bar{m}$, and $\epsilon_t^d = \psi (\log \xi_t^d - \log \bar{\xi}^d)$.

Linearizing the non-linear Phillips curve (PC) yields the Phillips-curve equation:

$$\pi_t = \kappa \hat{Y}_t, \quad (\text{AS})$$

where $\kappa \equiv \frac{\gamma}{1-\gamma}(\sigma^{-1} + \varphi)$ and $\varphi \equiv \frac{v_{NN}(\bar{N})\bar{N}}{v_N(\bar{N})} > 0$ (inverse Frisch elasticity).

Let us define the nominal money growth by $\mu_t \equiv \frac{M_t}{M_{t-1}}$. This definition implies that $m_t \equiv m_{t-1}\mu_t\Pi_t^{-1}$ or expressed with a log-linear approximated equation:

$$\hat{m}_t \equiv \hat{m}_{t-1} + \hat{\mu}_t - \pi_t. \quad (\text{MG})$$

A.3.8 - Approximated Equilibrium Definition

An approximated equilibrium is a collection of stochastic processes $\{\hat{Y}_t, \hat{i}_t, \pi_t, \hat{i}_t^r, \hat{S}_t, \hat{i}_t^l, \hat{m}_t\}$ that solve the IS curve (IS), the approximated asset pricing equations (AP1*, AP2*, AP3*), the LM curve (LM / ZLB), the Phillips-curve equation (AS), and the nominal money growth equation (MG) given the exogenous shocks $\{\hat{r}_t^e, \epsilon_t^d, \hat{\omega}_t\}$ and a policy specification for the sequence $\{\hat{\mu}_t\}$.

Appendix A.5 - Identifying Quasi-Money Demand

Table 5: Empirical Estimates

	β_0	β_1	β_2
Government Interest Rates	1.34	-0.88	-1.74
	(1.23, 1.45) ¹	(-1.00, -0.76) ¹	(-2.05, -1.43) ¹
	(1.19, 1.48) ²	(-1.03, -0.72) ²	(-2.15, -1.34) ²

¹95% and ²99% confidence intervals are in square brackets.